BASIC PROFESSIONAL TRAINING COURSE

Module I

Nuclear physics and reactor theory

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Background

In 1991, the General Conference (GC) in its resolution RES/552 requested the Director General to prepare 'a comprehensive proposal for education and training in both radiation protection and in nuclear safety' for consideration by the following GC in 1992. In 1992, the proposal was made by the Secretariat and after considering this proposal the General Conference requested the Director General to prepare a report on a possible programme of activities on education and training in radiological protection and nuclear safety in its resolution RES1584.

In response to this request and as a first step, the Secretariat prepared a Standard Syllabus for the Post-graduate Educational Course in Radiation Protection. Subsequently, planning of specialised training courses and workshops in different areas of Standard Syllabus were also made. A similar approach was taken to develop basic professional training in nuclear safety. In January 1997, Programme Performance Assessment System (PPAS) recommended the preparation of a standard syllabus for nuclear safety based on Agency Safely Standard Series Documents and any other internationally accepted practices. A draft Standard Syllabus for Basic Professional Training Course in Nuclear Safety (BPTC) was prepared by a group of consultants in November 1997 and the syllabus was finalised in July 1998 in the second consultants meeting.

The Basic Professional Training Course on Nuclear Safety was offered for the first time at the end of 1999, in English, in Saclay, France, in cooperation with Institut National des Sciences et Techniques Nucleaires/Commissariat a l'Energie Atomique (INSTN/CEA). In 2000, the course was offered in Spanish, in Brazil to Latin American countries and, in English, as a national training course in Romania, with six and four weeks duration, respectively. In 2001, the course was offered at Argonne National Laboratory in the USA for participants from Asian countries. In 2001 and 2002, the course was offered in Saclay, France for participants from Europe. Since then the BPTC has been used all over the world and part of it has been translated into various languages. In particular, it is held on a regular basis in Korea for the Asian region and in Argentina for the Latin American region.

In 2015 the Basic Professional Training Course was updated to the current IAEA nuclear safety standards. The update includes a BPTC text book, BPTC e-book and 2 "train the trainers" packages, one package for a three month course and one package is for a one month course. The" train the trainers" packages include transparencies, questions and case studies to complement the BPTC.

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Editorial Note

The update and the review of the BPTC was completed with the collaboration of the ICJT Nuclear Training Centre, Jožef Stefan Institute, Slovenia and IAEA technical experts.

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1 ATOMIC STRUCTURE OF MATTER

Learning objectives:

After completing this chapter, the trainee will be able to:

1. *Describe the terms element, atom, compound and molecule.*
2. *Define the atomic mass unit.*
3. *Define the relative atomic mass A_r.*
4. *Define the relative molecular mass M and calculate it from the chemical formula.*
5. *Calculate the mass of elements in a given mass of a compound.*

1.1 Elements, atoms, compounds and molecules

All matter is made up of basic components called the **elements**. These are the most basic substances and not even chemical changes can break them up into simpler substances.

> There are currently 112 known elements, 90 of which occur naturally, while the rest are man-made. Each element has a name and symbol which is derived from its Latin name: hydrogen – H, helium – He, lithium – Li, beryllium – Be, carbon – C, nitrogen N, oxygen – O, uranium – U, etc. These elements belong to those that can be found in nature in larger or smaller quantities. Two examples of synthetically produced elements are technetium – Tc and plutonium – Pu.

The smallest particle of an element to retain its chemical properties is called an **atom** (Greek *atomos* = indivisible). Atoms of the same element are identical to each other and atoms of different elements differ from one another in mass, size and inner structure. These varying properties of different atoms also result in the different chemical and physical properties of individual elements.

Two or more elements can bind together to make a **compound**. Compounds are formed when atoms of different elements combine. This set of atoms is called a **molecule** and is the smallest particle of a compound to retain its chemical properties. The molecule of a particular compound always contains a definite type and number of individual atoms. For instance, when hydrogen bonds with oxygen into water, an atom of oxygen (O) always combines with two atoms of hydrogen (H) into a molecule of water (H_2O). The chemical formula of a compound gives the atomic structure of its molecules.

1.2 Relative atomic and molecular mass

Since atoms are extremely small and extremely light, a special unit of mass was introduced - the **atomic mass unit** (u) which amounts to $1.66 \cdot 10^{-27}$ kg.

The atomic mass unit is defined as 1/12 of the mass of an atom of carbon-12.

The atomic mass of an element is usually expressed by the number of atomic mass units:

$$\text{Atomic mass} = A_r \text{ u}.$$

The number A_r is called the **relative atomic mass** and tells us how many times the atomic mass of an element is greater than the atomic mass unit. Each chemical element has an established relative atomic mass A_r. Information about relative atomic mass can be found e.g. in the periodic table (Appendix A). For example, for hydrogen $A_r = 1.0079 \approx 1$, for helium $A_r = 4.0026 \approx 4$, boron $A_r = 10.811 \approx 10.8$, carbon $A_r = 12.011 \approx 12$, oxygen $A_r = 15.999 \approx 16$, iron $A_r = 55.846 \approx 55.8$, uranium $A_r = 238.03 \approx 238$, etc.

The molecules of a compound are made up of the atoms of the elements which bind to form the compound. The mass of a molecule is the sum of the masses of the atoms it consists of. Molecular mass is expressed in a similar way as atomic mass - by the number of atomic mass units:

$$\text{Molecular mass} = M_r \text{ u}\ .$$

The number M_r is called the **relative molecular mass** of the compound and is equivalent to the sum of the relative atomic masses of the elements that make up the compound. The number and type of atoms that make up a compound is clear from its chemical formula.

Example:
Calculate the relative molecular mass and the mass of a molecule of water (H_2O).
Answer:
The chemical formula for water tells us it consists of two hydrogen atoms and one oxygen atom. Its relative molecular mass is therefore:

$$M_{rH2O} = 2 \cdot 1 + 16 = 18\ ,$$

which means that a molecule of water has a mass of:

$$m = 18 \text{ u} = 18 \times 1.66 \cdot 10^{-27} \text{ kg} = 3 \cdot 10^{-26} \text{ kg}.$$

One litre or one kg of water thus contains $1/(3 \cdot 10^{-26}) = 3.33 \cdot 10^{25}$ molecules of H_2O, specifically $3.33 \cdot 10^{25}$ oxygen atoms and $6.66 \cdot 10^{25}$ hydrogen atoms.

1.3 Exercises

1. Calculate the relative molecular mass of carbon dioxide.
2. How many boron atoms are there in a kilogram of boron?
3. How many hydrogen atoms and how many oxygen atoms are there in a kilogram of water?
4. What is the mass of hydrogen and the mass of oxygen in a

kilogram of water?

5. What is the mass of boron in 10 kg of boric acid, H_3BO_3?
6. How much oxygen must be added to 2 g of hydrogen to have all the hydrogen react with oxygen into water? How much water would we get?
7. How many uranium atoms are there in a cube with an edge length of 1 cm? (What additional information do you need?)

2 STRUCTURE OF THE ATOM

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Describe the structure of an atom.*
2. *Name the main characteristics of an electron.*
3. *Define the atomic number, Z.*
4. *Explain the terms positive and negative ion.*
5. *Define the binding energy of an electron.*
6. *Define the unit electron-volt.*
7. *Describe the energy levels of electrons in an atom.*
8. *Explain the ground state and excited state of an atom.*
9. *Explain the transition of an atom from an excited state to the ground state.*

2.1 Description of the atom

An atom is the smallest particle of an element to retain its properties, but this microscopically small particle (approx. 10^{-10} m) is composed of even smaller particles. The internal structure of an atom may be roughly compared to the solar system. Just as the planets orbit the (much heavier) sun, the atom has light particles, **electrons**, moving around a **heavy nucleus** (Figure 2.1). The atomic nucleus is ten thousand times smaller than the whole atom (the diameter of the nucleus is only 10^{-14} m) and yet the bulk (99.95%) of atomic mass is concentrated in the nucleus.

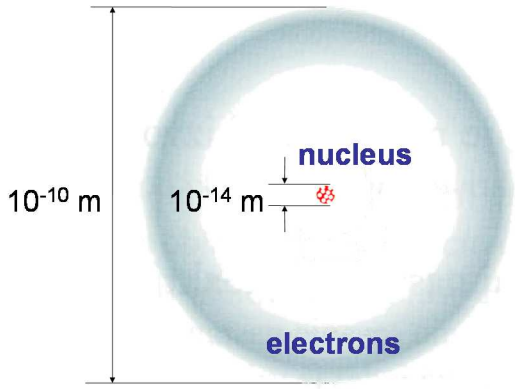


Figure 2.1: The structure of an atom (size of the nucleus is not to scale!).

There is an **attractive electrical force** between electrons and the nucleus, namely between the negatively charged electrons and the positively charged nucleus. The charge of an electron is:

$$e_0 = -1.6 \cdot 10^{-19} \text{ As} .$$

This charge is called the **elementary charge**.

Electrons are among the lightest particles we know, having a mass of approximately $9.1 \cdot 10^{-31}$ kg = 0.00055 u, which is nearly 2000 times less than the mass of the lightest hydrogen atom. Electrons move inside the atom as a sort of cloud which forms the outer boundary of the atom. This cloud is called the electron cloud and determines the chemical, mechanical, electrical and other properties of elements.

Every atom of the same element contains the same number of electrons, which differs for different elements. A hydrogen atom, for example, has one electron, a helium atom 2 electrons, an oxygen atom 8 electrons and so on. The number of electrons in an atom determines the chemical properties of the element and its position in the periodic system. The number of electrons in a neutral atom is called the **atomic number, *Z***: $Z = 1$ for hydrogen, $Z = 2$ for helium, $Z = 3$ for lithium, $Z = 92$ for uranium, etc.

Externally, atoms are as a rule electrically neutral, which means that the total negative charge of their electrons equals the positive charge of their nucleus. Hence, if a neutral atom has Z electrons, the positive charge of its nucleus is $+Z\,e_0$. Atoms that gain or lose a few electrons turn into a negative or positive **ion** (become charged). An atom that loses electrons changes into a positive ion (the positive charge of the nucleus is greater than the total negative charge of the electrons), whereas an atom that gains electrons changes into a negative ion (the charge of the electrons is greater than that of its nucleus).

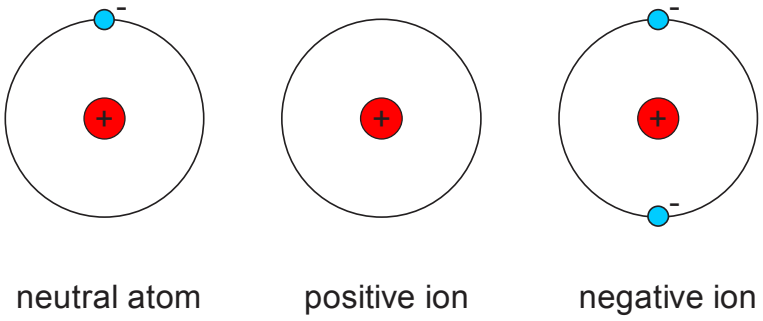


Figure 2.2: A neutral atom and a positive and negative ion of hydrogen.

2.2 Energy levels of electrons in the atom

Free electrons are electrons which leave the atom. When moving, free electrons have positive (kinetic) energy; when motionless, their energy is equal to zero. An electron which is bound in the atom must be supplied with energy to become free. This means that the energy of a bound electron is smaller than the energy of a free electron, i.e. it is less than zero. Hence, the energy of a bound electron or the **binding**

energy of an electron is always negative. The lower the binding energy (the more negative), the stronger the electron is bound.

Since atoms are extremely small, their energies are very much smaller than the energies we generally deal with. The energy unit used for measuring atomic energies is the **electron-volt** (**eV**), defined as the energy of an electron when accelerated by a voltage of 1 volt. One electron-volt is thus the product of the electron charge, i.e. its elementary charge $e_0 = 1.6 \cdot 10^{-19}$ As and a voltage of 1 V:

$$1 \text{ eV} = 1.6 \cdot 10^{-19} \text{ As} \cdot 1 \text{ V} = 1.6 \cdot 10^{-19} \text{ J}$$

Larger units are:

$$1 \text{ keV} = 10^3 \text{ eV} = 1.6 \cdot 10^{-16} \text{ J},$$
$$1 \text{ MeV} = 10^6 \text{ eV} = 1.6 \cdot 10^{-13} \text{ J}.$$

The binding energies of electrons in atoms range from a few eV in light atoms to a few keV in the heaviest atoms.

It turns out that bound electrons can only have the specific energies which are available in atoms. These discrete energies are called the **energy levels** of an electron in an atom. Each level can only be occupied by one electron. If the electrons in an atom occupy the lowest possible energy levels, the atom is said to be in its **ground state**. As soon as the electrons in the atom occupy higher energy levels, leaving lower levels empty, the atom is in an **excited state**. The atom does not remain in this state for long; in a short while, an electron from a higher energy level will fall into the empty lower energy level, giving off the energy difference in the form of a photon (the smallest packet of electromagnetic radiation) in the process. When this transition occurs between two levels close to each other in terms of energy, a photon of visible light is emitted; for large energy differences (~keV) a photon of ultraviolet light or X-radiation is emitted. X-radiation photons are also called **X-rays**.

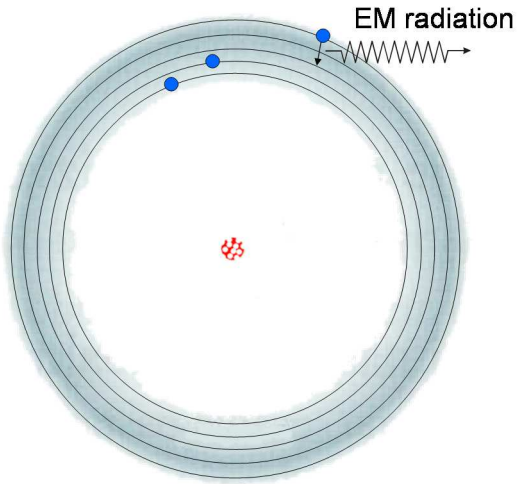


Figure 2.3: Transition of an atom from an excited state to the ground state.

3 ATOMIC NUCLEUS

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Describe the basic properties of protons and neutrons.*
2. *Define the mass number A and write the relationship between mass number, A, atomic number, Z, and the number of neutrons in the nucleus, N.*
3. *Define a nuclide and an isotope and describe their notation.*
4. *Explain the isotopic abundance of an element.*
5. *Describe the systematic arrangement of nuclei in a table.*
6. *Describe the energy states of a nucleus.*
7. *Define the binding energy of a nucleon.*
8. *Plot the binding energy of a nucleon as a function of mass number, A.*

3.1 Composition of the nucleus, nuclear forces

The atomic nucleus is made up of more basic particles which are called **nucleons**. There are two types of nucleons, **protons** and **neutrons**. Nucleons have nearly the same mass, approximately 1 u, and the main difference between them is their charge: protons carry a positive elementary charge $+e_0$ while neutrons are neutral. The properties of nucleons are given in Table 3.1.

Table 3.1: The basic properties of nucleons.

particle	charge	mass
proton (p)	$+e_0$	1.0072766 u
neutron (n)	no charge	1.0086654 u

Proton is actually just another name for a **hydrogen nucleus**. If a proton attracts a negative electron, we get a hydrogen atom. **Neutrons** are slightly heavier than protons. A free neutron is not stable.

With only positively charged protons in the nucleus (besides the neutral neutrons), strong electrical repulsive forces exist between them. Clearly, there must also be some attractive forces in the atomic nucleus, stronger than electrical forces, or the electrostatic charge would cause the nucleus to break apart. These forces are called **nuclear forces**.

> The nuclear force is an attractive force that equally attracts both types of nucleons. Unlike the electrical force it has a very short range (~10^{-15} m) and only acts on directly touching nucleons, whereas toward more distant nucleons, its intensity falls much more quickly than the repulsive electrostatic force between protons.

Since the charge carriers in the nucleus are protons which have a charge opposite to electrons, the number of protons in the nucleus must correspond to the number of electrons in the neutral atom. The **number of protons** in a nucleus is thus equal to the order number of an element in the periodic table and is called **atomic number, *Z***.

In addition to protons, every nucleus (except hydrogen) includes neutrons and the total **number of nucleons** is called the **mass number, *A***. Also used, although more rarely than atomic number, *Z*, and mass number, *A,* is the number of neutrons or the **neutron number**, ***N***, where:

$$N = A - Z\,.$$

An atom with a nucleus possessing a definite atomic and mass number is called a **nuclide**. A nuclide is fully defined by indicating the chemical element **X** and the nucleon numbers *Z*, *N* and *A*:

$${}^{A}_{Z}\mathrm{X}_{N}\,.$$

Examples: ${}^{1}_{1}\mathrm{H}_{0}$, ${}^{2}_{1}\mathrm{H}_{1}$, ${}^{4}_{2}\mathrm{He}_{2}$, ${}^{12}_{6}\mathrm{C}_{6}$, ${}^{27}_{13}\mathrm{Al}_{14}$, ${}^{60}_{27}\mathrm{Co}_{33}$, ${}^{137}_{55}\mathrm{Cs}_{82}$, ${}^{238}_{92}\mathrm{U}_{146}$ …

Since the atomic number, *Z*, is uniquely associated with the chemical name of element X, it may be omitted in the nuclide notation. We can also leave out the neutron number, *N*, because it equals the difference between the mass number and the atomic number. It follows that a nuclide is fully and uniquely defined just by indicating the chemical element and mass number:

$${}^{A}\mathrm{X}\,.$$

Hence, the above examples can be written in a shorter way: ^{1}H, ^{2}H, ^{4}He, ^{12}C, ^{27}Al, ^{60}Co, ^{137}Cs, ^{238}U. When referring to nuclides with the full name of the element, we should add the mass number, e.g. helium-4, carbon-12, aluminium-27, uranium-238.

3.2 Isotopes

Nuclides that share the same atomic number, *Z*, are called **isotopes**. Since the number of protons in the nucleus defines the chemical element to which nuclide belongs, **isotopes** are species of the same chemical element. Isotopes have the same number of protons (atomic number *Z*) but vary in the number of neutrons (neutron number *N*) or the total number of nucleons (mass number *A*). A few examples: the hydrogen isotope ^{1}H has one proton, isotope ^{2}H has a proton and a neutron, and isotope ^{3}H has a proton and two neutrons The helium isotope ^{3}He has two protons and a neutron, ^{4}He has two protons and two neutrons, the carbon isotope ^{12}C has 6 protons and 6 neutrons, ^{13}C has 6 protons and 7 neutrons, etc. Among the naturally occurring

elements, uranium has the heaviest nucleus (Z = 92). Its best known isotopes are ^{238}U and ^{235}U.

Isotopes also have an identical number of electrons (being the same chemical element) and thus the same chemical properties, but they differ in mass number as well as mass; their atomic nuclei have different masses. The lightest element, hydrogen, has two isotopes in nature. In addition to common hydrogen with a mass number 1, also called light hydrogen (H), there is **heavy hydrogen** or **deuterium** with a mass number of 2. A deuterium atom is denoted D, and its nucleus or *deuteron* is denoted d. There is a third isotope of hydrogen, **tritium** (T or t), with a mass number of 3 (one proton and two neutrons); however, it is not stable in nature (it is produced artificially).

In nature, most chemical elements consist of different isotopes. These occur in fixed relative proportions which are called the **isotopic abundances** of the element. Examples:

Hydrogen: ^{1}H – 99.985%, ^{2}H – 0.015% .
Boron: ^{10}B – 19.8%, ^{11}B – 80.2% .
Aluminium: ^{27}Al – 100% .
Iron: ^{54}Fe – 5.8%, ^{56}Fe – 91.72%, ^{57}Fe – 2.2%, ^{58}Fe – 0.28% .
Uranium: ^{234}U – 0.0055%, ^{235}U – 0.72%, ^{238}U – 99.27% .

3.3 Table of nuclides

Of around 3000 known nuclides only 237 are stable. All other nuclei sooner or later undergo one or several nuclear changes (radioactive decays) and change into stable nuclei.

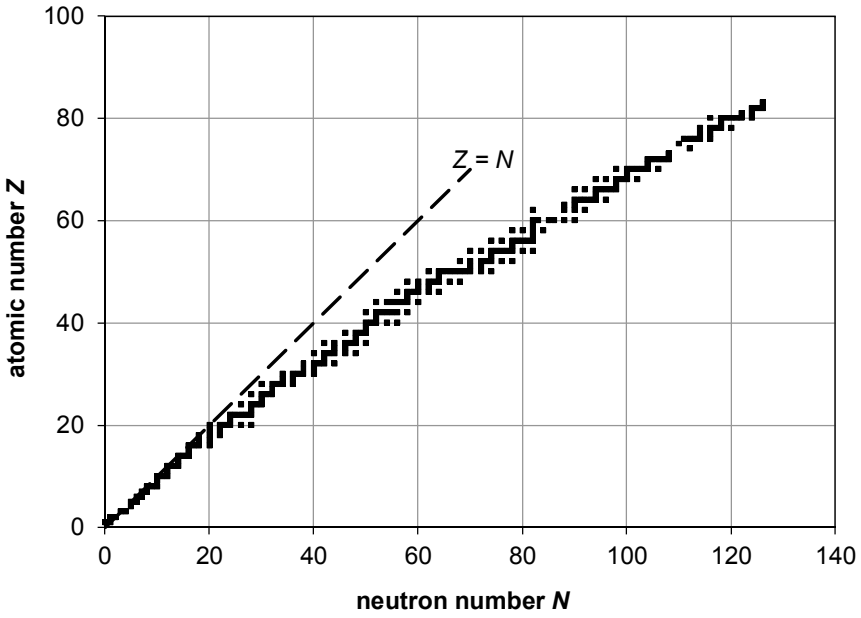


Figure 3.1: Table of the stable nuclides.

In Figure 3.1 the stable nuclides are arranged in a chart by placing the neutron number, $N = A - Z$, on the horizontal axis and the atomic number, Z, on the vertical axis. **Isotopes** thus lie in the same horizontal row.

The region of stability is determined by the ratio between protons and neutrons which matches the lowest internal energy of a nucleus. It is energetically favourable to have one neutron to each proton in the nucleus. This is why light stable nuclides feature the same number of protons and neutrons, i.e. they lie on the line $N = Z$, e.g. ${}^{4}_{2}\mathrm{He}_2$, ${}^{10}_{5}\mathrm{B}_5$, ${}^{12}_{6}\mathrm{C}_6$, ${}^{14}_{7}\mathrm{N}_7$, ${}^{16}_{8}\mathrm{O}_8$ etc. In heavier nuclides, an important role is played by the electrostatic repulsion between protons, which is why the energy of a nucleus with a large A would increase less by adding two neutrons than by adding a proton and a neutron. Thus in heavy nuclides neutrons outnumber protons ($N >> Z$): the nucleus of the heaviest stable lead isotope, ${}^{208}\mathrm{Pb}$, has 82 protons and 126 neutrons.

Like electrons in the atom, nucleons in the nucleus occupy different energy levels. As a rule all nucleons have the lowest possible energy level and this is when the nucleus is said to be in the **ground state**. When not all the nucleons occupy the lowest energy level, the nucleus is in an **excited state**. Like atoms, the energy levels of nuclei have only specific discrete values of energy. The differences between the energy levels of the nucleus are of the order of magnitude of MeV and thus considerably larger than for electrons in an atom. The distribution of energy levels is characteristic for each nuclide: in heavier nuclei they are closer together than in light nuclei (Figure 3.2).

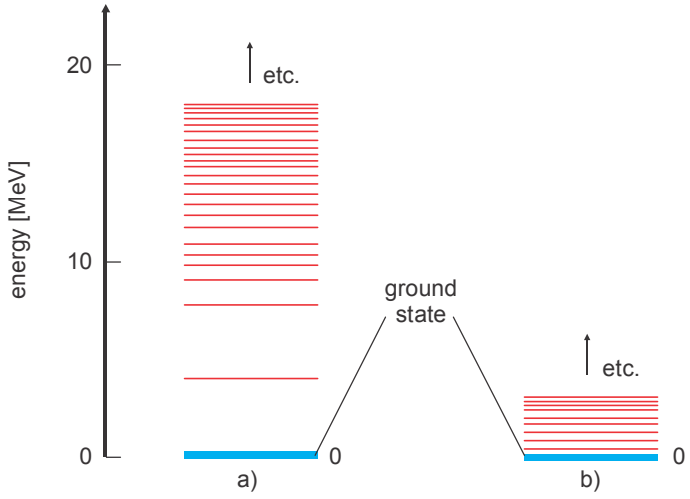


Figure 3.2: Energy levels in: a) light nuclei, b) heavy nuclei.

3.4 Binding energy of a nucleon in the nucleus

The energy of a free nucleon (e.g. proton) which is motionless equals zero. To release a nucleon bound in the nucleus from the nucleus, it has to be supplied with a certain energy, hence the energy of a bound nucleon is less than the energy of a free nucleon. The **binding energy** of a nucleon is the energy it needs to be supplied with to release it from the nucleus. **Binding energy** is always negative.

For the lightest nucleus (hydrogen), the binding energy equals zero, since the proton is not bound to any other nucleon. As the number of nucleons in the nucleus increases, so does the absolute value of the binding energy of individual nucleons, i.e. it becomes increasingly negative. Binding energy is the most negative (around -8.5 MeV per nucleon) in medium-heavy nuclei (with a mass number between 40 and 100). This means that medium-heavy nuclei are the most stable; nucleons are most firmly bound in these nuclei.

The plot of binding energy per nucleon with mass number (Figure 3.3) then gradually rises towards heavy nuclei. These are the less stable the more nucleons they contain.

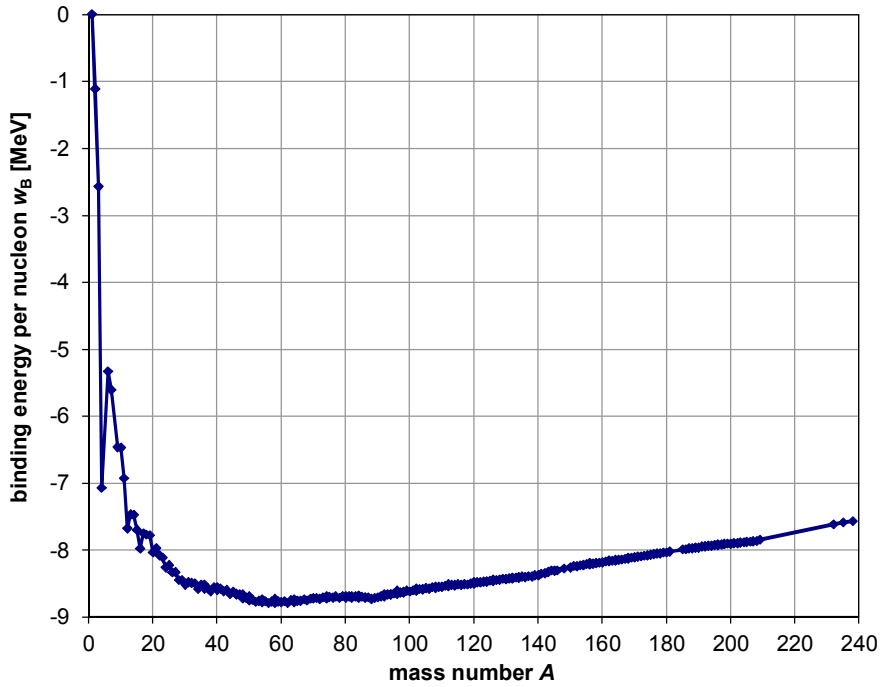


Figure 3.3: Nucleon binding energy as a function of mass number.

Based on the binding energy curve, we can conclude that combining two light nuclei into a medium-weight nucleus lowers the binding energy of individual nucleons; a medium-weight nucleus has less internal energy than the light ones together and the difference in energy is released during the reaction. Thus, when light nuclei combine, energy is released. This nuclear reaction is called **fusion** and it takes place in the Sun and other stars. Humanity has so far only learned how to use it for military purposes, but intensive research on

controlled nucleus fusion (to generate energy) is underway.

As in the fusion of two light nuclei, splitting heavy nuclei into two medium-heavy nuclei also lowers the binding energy per nucleon, which means that splitting heavy nuclei likewise releases energy. This process is called nuclear **fission** and is currently the main source of nuclear energy used for peaceful purposes.

4 RADIOACTIVITY

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Describe the phenomenon of radioactive decay.*
2. *Describe the random nature of radioactive decay.*
3. *Define the half-life, average lifetime and the decay constant of a radioactive nuclide.*
4. *Define the activity of a radioactive nuclide.*
5. *Write the equation for the decay of activity with time.*
6. *Calculate using the exponential law of radioactive decay.*
7. *Explain the terms short-lived, long-lived and stable nuclide.*
8. *Describe alpha, beta and gamma radioactive decay and give the basic properties of these types of radiation.*

4.1 Unstable nuclei, radioactive decay

Nuclides located outside the range of stability in Figure 3.1 are unstable. Unstable nuclides change into stable nuclides by undergoing internal changes and emitting particles and energy. The process of internal change is called **radioactive decay** and unstable nuclides are called **radioactive** nuclides or **radionuclides**. We know of around 3000 radionuclides, 100 of which occur naturally while the rest are produced synthetically (by nuclear reactions).

In radioactive decay, the internal energy of a nucleus decreases and the energy difference is carried away by electromagnetic radiation or particles which are emitted by the nucleus with great kinetic energy. These high-energy particles and electromagnetic radiation which are produced in the decay of unstable nuclei are collectively called **radioactive radiation**.

Radioactive decay takes place of its own accord, spontaneously. It is very important that the rate and the manner in which a certain unstable nucleus decays depends solely on its internal structure; **decay cannot be influenced from outside**. Radioactive decay is not affected by changes in pressure, temperature, chemical change and so on.

Radioactivity is a **random process**. This means that we cannot predict the moment when a nucleus will decay. If we have several identical unstable nuclei, it is likely that some of them will undergo decay as soon as they are formed, others a little later, while others still will remain in an unstable state for some time to come. However, in the case of a large number of identical nuclei we can predict the **probability** of decay for a certain proportion of nuclei per time unit.

Some types of radioactive nuclei on average decay faster than others (are more unstable) and their number decreases faster with time. It

turns out that each unstable nuclide has its own characteristic probability of decay per time unit which is constant and independent of external factors. The probability of decay is accordingly called the **decay constant, λ** and has units s^{-1}.

The number of radioactive nuclei decreases **exponentially** with time. The law of radioactive decay is best described by the concept of half-life.

The **half-life $t_{1/2}$** is defined as the **time in which the number of radioactive nuclei decreases by half**. The number of radioactive nuclei is further halved with each passing of a half-life. If there are initially ($t = 0$) n_0 radioactive nuclei, there will be $n_0/2$ left after one half-life ($t = t_{1/2}$), a half of $n_0/2$ or $n_0/4$ after two half-lives ($t = 2\ t_{1/2}$), $n_0/8$ after three half-lives ($t = 3\ t_{1/2}$) and so on (Figure 4.1).

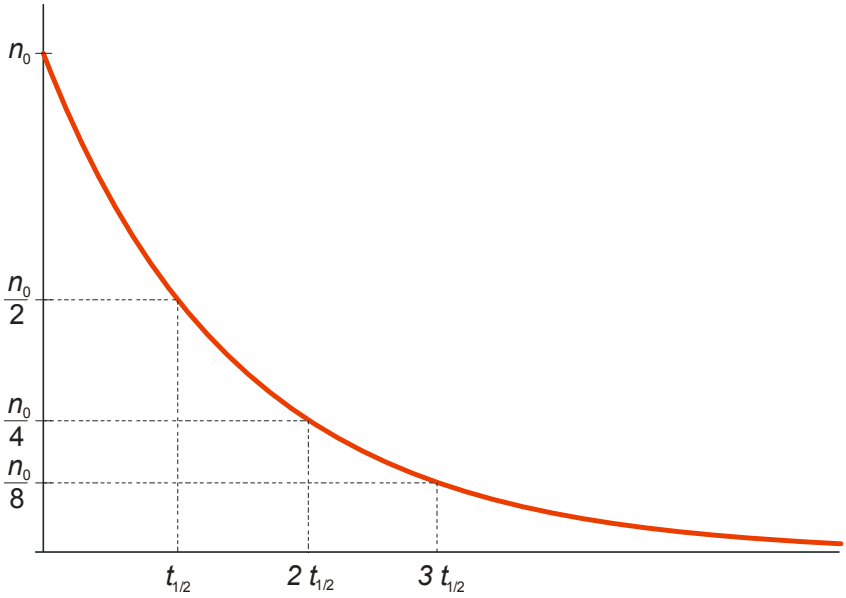


Figure 4.1: The exponential law of radioactive decay.

The exponential law of radioactive decay is generally written in the following way:

$$n(t) = n_0 \cdot 2^{-\frac{t}{t_{1/2}}},$$

where n_0 is the number of radioactive nuclei at the beginning (at time $t = 0$), $n(t)$ is their number at time t, and $t_{1/2}$ is the half-life of the given nuclide.

The decay constant, λ, and half-life, $t_{1/2}$, are related by the following equation:

$$t_{1/2} = \frac{0.693}{\lambda}.$$

The relevant half-life can be found in tables of nuclides (e.g. Appendix B). Nuclides with a high probability of decay have a large decay constant and a short half-life. Such nuclides decay quickly, i.e their number decreases quickly. They are also called **short-lived** nuclides. In contrast, nuclides with a small decay constant and a long half-life decay slowly or their number decreases slowly, which is why they are called **long-lived** nuclides. Clearly, the most long-lived nuclides are stable nuclides with a decay constant of zero (Figure 4.2).

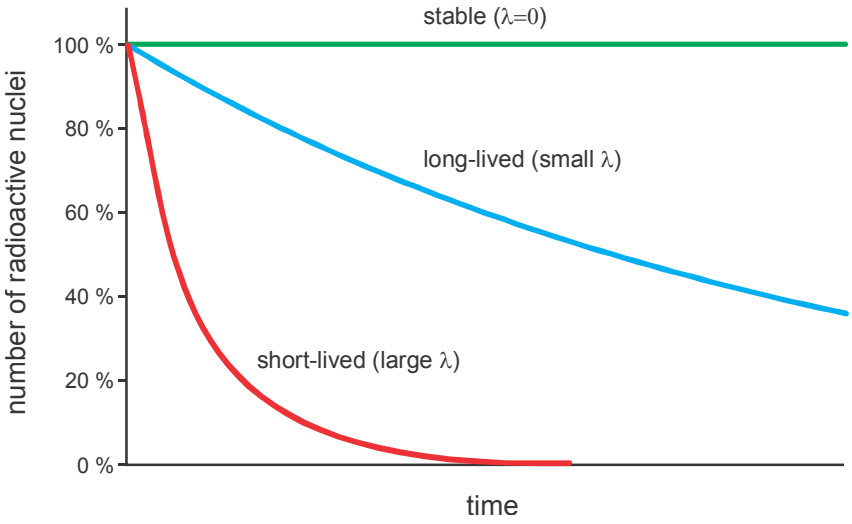


Figure 4.2: Comparison of nuclides with different decay constants.

Sometimes we also refer to the **mean lifetime, τ,** of a radioactive nucleus, which is defined as the average time from its formation to its decay. The average lifetime is 44% longer than the half-life, which can be written as:

$$\tau = 1.44\, t_{1/2}\ .$$

4.2 Activity

What interests us most about a radioactive source is not so much how many radioactive nuclei it contains but rather how many of them decay in a unit of time, e.g. per second. As each disintegration is combined with emission of radiation, we want to know how many particles or how much electromagnetic radiation a source emits in unit time.

The number of radioactive nuclei disintegrating per unit of time is called the **source activity** ($\mathcal{A}c$). The source activity is obviously higher if the source contains more radioactive nuclei (large n) and if these decay more quickly (large decay constant). Activity is simply the product of the number of radioactive nuclei and the nuclide decay constant (the probability of decay):

$$\mathcal{A}c = \lambda \cdot n \, .$$

Since λ is a constant, i.e. is independent of time, the radioactive source activity changes with time in the same way as the number of radioactive nuclei – decreasing exponentially:

$$Ac(t) = Ac_0 \cdot 2^{-\frac{t}{t_{1/2}}} \quad ,$$

where $\mathcal{A}c_0$ is the initial source activity and $t_{1/2}$ is the half-life.

The unit of activity is 1 disintegration/s and is called a **becquerel** (Bq):

1 Bq = 1/s .

This is a very small unit, considering that even a minor radioactive source has an activity of several million Bq. Sources with an activity of a few ten kBq are usually below the legal exemption limit, meaning they are not officially regarded as radioactive sources. The old unit of activity, 1 Ci (**curie**), was defined as the activity of 1 gram of radium-226 and amounts to $3.7 \cdot 10^{10}$ disintegrations per second:

1 Ci = $3.7 \cdot 10^{10}$/s = $3.7 \cdot 10^{10}$ Bq = 37 GBq .

Smaller units are:

1 mCi = 10^{-3} Ci = $3.7 \cdot 10^{7}$/s = 37 MBq ,
1 μCi = 10^{-6} Ci = $3.7 \cdot 10^{4}$/s = 37 kBq .

Rather than the total source activity, we are often more interested in the **specific activity, *ac***, i.e. activity per unit mass (more rarely per unit volume). This is calculated by dividing the total activity, $\mathcal{A}c$, by the mass of substance, m:

$$ac = \mathcal{A}c \, / \, m$$

Example:
A barrel weighing 460 kg contains 69 MBq of the radionuclide ^{137}Cs. Calculate its specific activity.
Answer:
Its specific activity is:

ac = 69 MBq / 460 kg = 0.15 MBq/kg = 150 kBq/kg = 150 Bq/g.

In the following sections, we are going to look at different types of radioactive decay. The type of decay a certain unstable nucleus undergoes is determined by the type of radiation given off by the nucleus as it decays; the most common types of decay are **alpha**, **beta** and **gamma** decay. Just like the decay rate, the decay type cannot be influenced externally; it is determined by the internal structure of the nucleus.

4.3 Alpha decay

As a rule, alpha active nuclei are heavy nuclei which are unstable due to the strong electrostatic repulsion between the numerous protons. Such nuclei reduce their internal energy by emitting an alpha particle, i.e. a helium nucleus. An example of alpha decay is radium-226:

$$^{226}_{88}\mathrm{Ra} \rightarrow {}^{222}_{86}\mathrm{Rn} + \alpha$$

By emitting an alpha particle the nucleon loses two protons and two neutrons so that its mass number is 4 less and its atomic number 2 less. Alpha decay is schematically written like this:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$$

Alpha particles have an energy of a few MeV. They are stopped by just a very thin layer of matter such as a sheet of paper or the skin's surface. In air, their range is a few centimetres apart from uranium isotopes, important alpha emitters in nuclear technology include the isotopes of transuranium elements (plutonium, americium, etc.). In daily life, an important radionuclide is the gas radon (^{222}Rn) which is an alpha emitter and responsible for the bulk of the natural human exposure to radiation.

4.4 Beta decay

Beta active nuclei are nuclei with a ratio between the number of protons and neutrons which lies outside the stability range. Beta nuclei thus have an excess of either neutrons or protons.

Depending on the type of excess nucleons in the nucleus, there are different subtypes of beta decay: nuclei with too many neutrons undergo **beta minus** decay and nuclei with too many protons undergo **beta plus** decay or, alternatively, **electron capture**.

Nuclei with an **excess of neutrons** undergo β^- decay. In this type of decay, the number of neutrons in the nucleus falls by one, the number of protons rises by one, and the nucleus emits a beta minus particle which is essentially an electron with large kinetic energy. In addition, beta decay produces a neutrino, a particle with no mass which virtually does not react with matter or the environment, so it will not be mentioned again.

An example of a β^- decaying nuclide is:

$$^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \beta^{-}$$

Beta minus decay increases the number of protons in the nucleus by one and lowers the number of neutrons by one, producing an element with an atomic number which is higher than originally by one. However, its mass number remains the same. Beta minus decay is schematically written like this:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + \beta^{-}$$

Beta minus decay can also be described by the transformation that takes place in the nucleus:

$$\mathrm{n} \rightarrow \mathrm{p} + \mathrm{e}^{-}.$$

This decay may also occur in nature, since a free neutron is not stable.

Beta particle energies range from a few tens of keV to MeV. They are more penetrating than α particles. Their range is a few metres in air and a few millimetres in metals. Nuclear reactors often produce nuclei with an excess of neutrons, so the great majority of radionuclides relevant to nuclear technology disintegrate by beta minus decay. The most important ones include ^{3}H, ^{16}N, ^{60}Co, ^{90}Sr, ^{131}I, ^{137}Cs, etc.

In β^{+} decay, the number of neutrons in the nucleus increases by one, the number of protons decreases by one and the mass number is unchanged. A beta plus particle or **positron** is produced. This particle closely resembles an electron except that it has a positive electrical charge. When a positron comes into contact with a common electron, both particles disappear, producing two photons with 511 keV energy. This phenomenon is called **annihilation**.

An example of a nuclide disintegrating by β^{+} decay is sodium-22:

$$^{22}_{11}\mathrm{Na} \rightarrow {}^{22}_{10}\mathrm{Ne} + \beta^{+}$$

Beta plus decay is schematically written like this:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \mathrm{e}^{+}$$

or described by the transformation occurring in the nucleus:

$$\mathrm{p} \rightarrow \mathrm{n} + \beta^{+}.$$

This transformation cannot take place independently, since the free proton (hydrogen nucleus) is stable.

4.5 Gamma decay

The nucleus resulting from alpha or beta decay is very often in an excited state. Like an atom which passes from an excited state into the ground state, emitting a photon of visible or ultraviolet light or X-radiation, an excited nucleus is also unstable and usually very quickly passes into the ground state. The excess of energy on its transition into a lower or the ground energy state is carried away by the photon emitted by the nucleus. Of course, the photon's energy corresponds to the difference between the energy states of the nucleus and is of the order of magnitude of MeV. This transition between excited states is called **gamma decay** and the photons produced in the process are called **gamma rays.** They normally have considerably higher energy than X-rays which is why they are considerably more penetrating. They are likewise significantly more penetrating than alpha or beta particles. Gamma decay is relatively the mildest form of radioactive decay, since the nucleus merely "cools" whereas its internal composition is unchanged.

The excited nucleus which de-excites by gamma decay usually decays in such a short time that it is not really deemed an independent nuclide. Instead, the gamma rays are attributed to the nucleus that underwent alpha or beta decay. Such an example is ^{60}Co, which decays by β^- decay to ^{60}Ni and simultaneously emits gamma rays with energies of 1173 keV and 1332 keV. These gamma rays ar emitted when the excited nucleus ^{60}Ni de-excites, but the lifetime of this excited nucleus is so short that it is unknown (Figure 4.4).

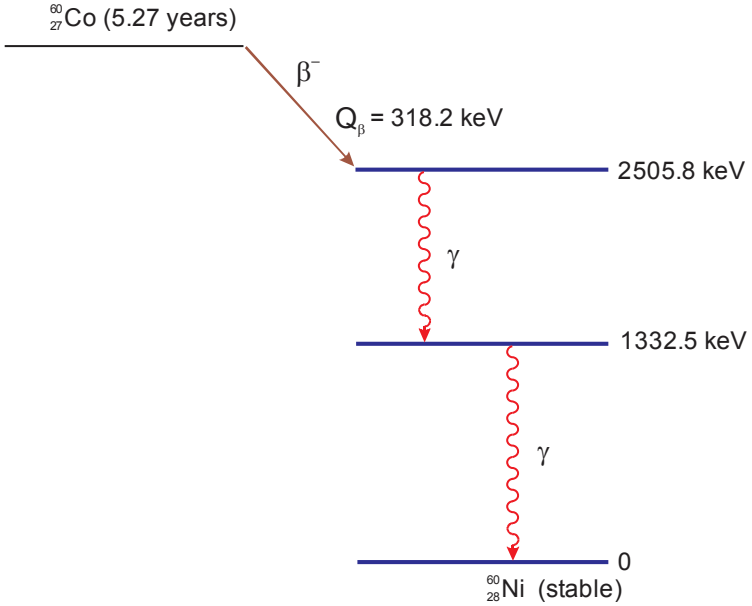


Figure 4.4: Decay scheme for ^{60}Co.

However, some excited nuclei are stable enough to exist in the excited state for a definite time. Such a nucleus is called a **nuclear isomer**.

An example is the nuclear isomer of barium-137 (denoted $^{137}Ba^{*}$ or ^{137m}Ba) with a half-life of 2.6 minutes. Gamma decay of ^{137m}Ba can be written as:

$$^{137m}_{56}Ba \rightarrow {}^{137}_{56}Ba + \gamma$$

or schematically:

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$$

Gamma decay thus involves no change in proton number, Z, or neutron number, N (and clearly no change in the total number of nucleons, i.e. mass number A).

4.6 General facts about radioactive decay and radiation

The table in Figure 3.1 shows all the stable nuclides. If this table is completed with the unstable or radioactive nuclides, we get Figure 4.5.

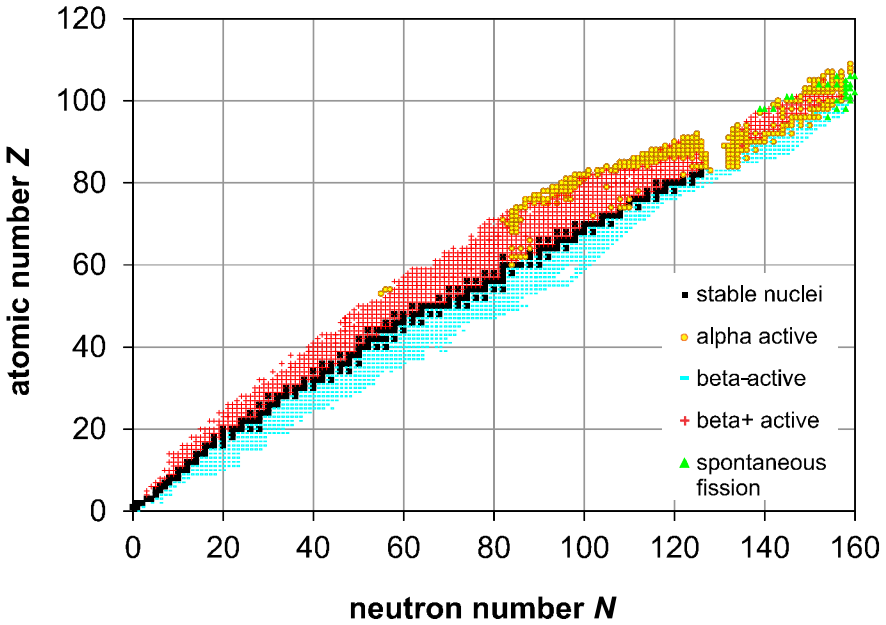


Figure 4.5: Table of nuclides ($t_{1/2}$ > 1 ms).

There are other, much more detailed tables of nuclides which provide a number of data for each nuclide.

Alpha and beta decay change the structure of the nuclei – this means that the nuclei which belonged to a particular element change into the nuclei of another element. It should be emphasized that this involves **spontaneous** nuclear changes, in other words changes which happen

by themselves and cannot be influenced from outside.

In any change of the atomic nucleus, the **number of nucleons** and **electric charge are conserved** as evident from the schematic "equations" for each type of decay:

- alpha decay: ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}$,
- beta decay: ${}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e}$,
- gamma: ${}^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X} + {}^{0}_{0}\gamma$,

A defined initial nucleus and type of decay therefore allow us to work out what nucleus will result from the decay.

Other characteristics of individual types of decay:

- Heavy nuclei undergo α decay.
- Nuclei with an excess of neutrons undergo β^- decay.
- Nuclei with an excess of protons undergo β^+ decay or electron capture (ε).
- α and β decays usually lead to a nucleus in an excited state which de-excites by γ decay.
- The excited state of the resulting nucleus is usually so short-lived that the γ rays are attributed to the precursor nucleus.

The types of radiation produced by radioactive decay are collected in Table 4.1 which gives the main characteristics of individual radiations.

Table 4.1: Radiation characteristics.

name	symbol	characteristic	mass	charge	penetration depth
alpha	α	^{4}He nuclei	≈ 4 u	+2 e_0	least penetrating radiation
beta	β^- β^+	electrons positrons	0.00055 u	$-e_0$ $+e_0$	more penetrating than α
gamma	γ	EM radiation	–	0	more penetrating than β
neutron	n	nucleon	≈1 u	0	more penetrating than β

Neutrons generally do not result from radioactive decay but from nuclear reactions (in a reactor). This is discussed in Chapter 5.

4.7 Exercises

1. The radionuclide ^{131}I has a half-life of 8 days. What will its activity be after 16 days, if its initial activity was 16 MBq?

2. The radionuclide ^{16}N has a half-life of 7 s. Its initial activity is 96 GBq/m^3. How long does it take for its activity to fall to 3 GBq/m^3?
3. How many half-lives must pass for the activity to fall a) below 1%, b) below 0.1% and c) below 10^{-6} of the initial activity?
4. The initial activity of a sample was 24 kBq. After 52 minutes, it dropped to 1.5 kBq. What is the half-life of the radionuclide in the sample?
5. A radiographic source of ^{60}Co ($t_{1/2}$= 5.3 years) was made 16 years ago. What was its (approximate) initial activity if it is now 15 TBq?
6. If there are 1000 neutrons in a cubic metre of air, how many neutrons will remain after 1 day if the half-life of a free neutron is 10 minutes?
7. The current source activity of ^{131}I ($t_{1/2}$ = 8 days) is 0.25 MBq. Calculate its activity 32 days ago.
8. A sample was irradiated in the reactor until its activity reached 24 MBq. Twenty-four hours later its activity was down to 3 MBq. What is the half-life of the radionuclide in the sample?
9. A radon concentration of 125 Bq/m^3 was measured in the air. What is the total activity of radon in a room with an area of 6 x 4 m and a height of 2.2 m?

5 NUCLEAR REACTIONS

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Explain nuclear reactions.*
2. *Name the two key conservation laws of nuclear reactions.*
3. *Define the terms exoergic and endoergic nuclear reaction.*
4. *List the reactions of neutrons with matter.*
5. *Write down some important nuclear reactions.*
6. *Define neutron flux and the reaction cross section.*

5.1 General description of nuclear reactions

A nuclear reaction is a process in which atomic nuclei react with radiation or particles. The most common radiation or particles capable of causing nuclear reactions are neutrons, and in some exceptional cases also alpha particles or high-energy photons.

Nuclear reactions thus very often involve **nuclear changes**. Atomic nuclei also change in radioactive decay but there is a fundamental difference between nuclear reactions and decay: a nuclear reaction can be **controlled,** induced, accelerated, slowed down and stopped, whereas radioactive decay cannot be influenced from outside.

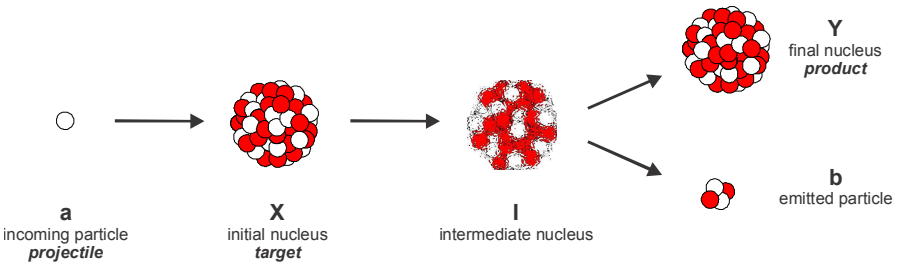


Figure 5.1: Course of a nuclear reaction.

The incoming particle (usually a neutron) is called a **projectile** (denoted a) and the irradiated substance is called the **target** (marked X). Some projectiles collide with target nuclei and induce a nuclear reaction. This first produces an **intermediate nucleus** (I) which practically instantly breaks down into a **final nucleus** (Y) and an **emitted particle** (b). The nuclear reaction process can thus be written as:

$$a + X \rightarrow I \rightarrow Y + b$$

or in short:

$$X + a \rightarrow Y + b\ .$$

The established notation for nuclear reactions is:

$$X\ (a,\ b)\ Y\ .$$

The first letter in brackets denotes the incoming particle and the second represents the emitted particle. The incoming and emitted particles determine the type or family of nuclear reactions, which often share similar characteristics though the target and initial nuclei differ.

Like radioactive decay, nuclear reactions are subject to conservation laws:

1. **Conservation of electric charge.** The total electric charge of all reaction participants before a reaction is the same as after the reaction.
2. **Conservation of the number of nucleons.** The number of nucleons (protons and neutrons) before the reaction is equal to the number of nucleons after it. In a nuclear reaction, the proton number can only increase at the expense of a reduced neutron number (or the other way round).

As an example, take the nuclear reaction between beryllium nuclei and alpha particles:

$$^{9}_{4}\mathrm{Be} + {}^{4}_{2}\alpha \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}\,.$$

This results in a nucleus of the carbon isotope ^{12}C and a neutron. Before the reaction, we had a total of 13 nucleons (9 in the beryllium nucleus and 4 in the alpha particle), and afterwards we again have 13 nucleons: 12 in the carbon nucleus and a free neutron. Charge is conserved along with the atomic number. Before the reaction the sum of the atomic numbers is 6 and after the reaction it is likewise 6.

The above nuclear reaction is written more briefly as:

$$^{9}_{4}\mathrm{Be}(^{4}_{2}\alpha,\ {}^{1}_{0}\mathrm{n})^{12}_{6}\mathrm{C}$$

or

$$^{9}\mathrm{Be}\ (\alpha,\ \mathrm{n})\ ^{12}\mathrm{C}$$

and belongs to the family of (α, n) reactions.

Based on whether energy is released or not in the nuclear reaction, reactions are either exoergic or endoergic.

A reaction is **exoergic** if accompanied by a release of energy. In an **endoergic reaction** energy is consumed, i.e. energy must be supplied for the reaction to take place. This means that a projectile must have a certain kinetic energy to induce a reaction. The **threshold of an endoergic reaction** is the minimum projectile energy required to

induce the reaction.

5.2 Reactions with neutrons

Neutrons have no electric charge, so they can easily approach atomic nuclei and enter them. This is why neutrons can cause nuclear reactions much more easily than other particles. A nuclear reactor is an extremely powerful source of neutrons in which neutron-induced reactions predominate.

There are two major families of neutron reactions: **scattering** and **absorption**. Scattering is essentially neutrons bouncing off nuclei. This does not produce new particles or nuclei but it does involve a transfer of energy. In absorption, a nucleus captures a neutron, which leads to a **nuclear change**.

Scattering

Scattering involves neutrons colliding with nuclei and bouncing off them. If a parallel beam of neutrons meets a group of atoms, individual neutrons bounce off atoms at different angles and the previously parallel beam disperses in different directions. The established term for this dispersal is **scattering**.

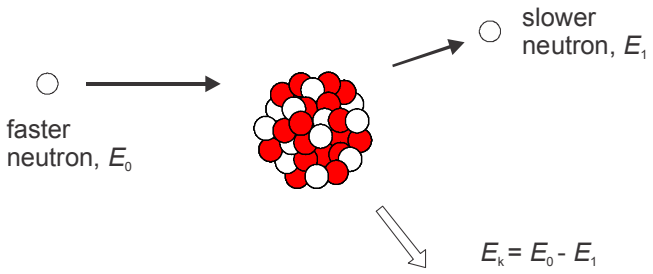


Figure 5.2: Neutron scattering.

An atomic nucleus is (nearly) motionless before colliding with a neutron but starts moving after the collision, much like a previously still billiard ball starts moving when hit by another ball. The nucleus moves as a result of taking some of the neutron's kinetic energy on collision. Consequently this means that the neutron's energy or speed has been reduced. Scattering is thus a process in which neutrons **slow down**. Slowing down is most effective when neutrons are scattered on light nuclei.

As a rule, neutrons are born in nuclear reactions (e.g. by nuclear fission) and have kinetic energy of one or several MeV. Such neutrons are called **fast neutrons**. Sequential scattering on atomic nuclei reduces the energy of neutrons step by step until it equals the energy of thermal motion of atoms in matter, which amounts to a few hundredths of an eV. Such neutrons are called **thermal** neutrons. The slowing-down process of neutrons is called neutron **moderation** or

thermalization.

Absorption

If a neutron reacts with a nucleus without scattering, the nucleus **absorbs** the neutron. This brings about a nuclear change. The most common absorption reaction is **radiative capture** or **(n, γ)** reaction.

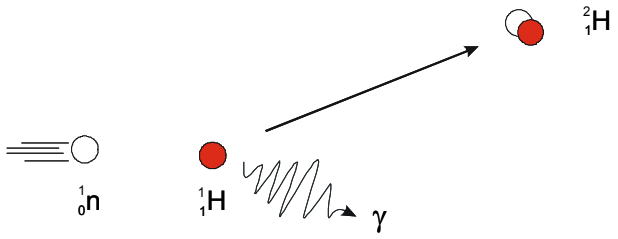


Figure 5.3: An example of radiative capture.

Alternatively, neutron capture can involve emitting some other particle, most frequently an alpha particle or proton, in other words an (n, α) or (n, p) reaction.

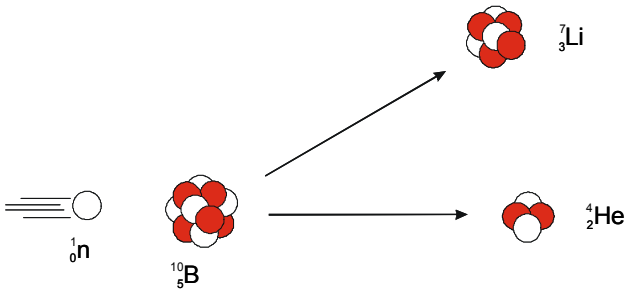


Figure 5.4: Example of an (n, α) reaction.

The third possible absorption reaction is nucleus splitting or fission. The whole of the next chapter is devoted to fission due to its importance for nuclear technology.

Finally, very high neutron energies may lead to reactions which involve emitting several particles, e.g. (n, 2n), (n, np), etc.

5.3 Rate of nuclear reactions

Besides the nature of nuclear reactions, i.e. which nuclei interact with neutrons and what becomes of them, we are generally also interested in the number of nuclear reactions that occur in matter in a particular period of time. This rate is proportional to the intensity of neutron radiation and the probability of a reaction on a certain nucleus. Neutron intensity is described by one quantity, the probability of a reaction on the nucleus by another, and the rate of reaction is proportional to the product of the two.

Intensity of neutron radiation – flux

The intensity of neutron radiation is defined as the number of neutrons per unit area in unit time:

$$\Phi = \frac{\text{number of neutrons}}{\text{time} \cdot \text{area}} \qquad [\text{cm}^{-2}\ \text{s}^{-1}] \ .$$

The quantity Φ is called **neutron flux**. Typical neutron flux values in a reactor range from 10^{12} cm^{-2} s^{-1} to 10^{14} cm^{-2} s^{-1}. The greater the neutron flux, the more nuclear reactions take place in a unit of time.

Probability of reaction on a target nucleus – cross section

The probability of a nucleus reacting with a neutron is described by a physical quantity called the **cross section**. The cross section σ may also be understood as the effective cross sectional area of a nucleus "seen" by a beam of particles. However, this geometrical comparison should not be taken too literally – cross section values change markedly from one nucleus to another, and also relative to neutron energy. The cross section has the dimension of surface area; as its typical values are very small, a special unit of area is introduced, the **barn**.

$$1 \text{ barn} \equiv 1 \text{ b} = 10^{-24} \text{ cm}^2 \ .$$

The rate of nuclear reactions in a target exposed to a neutron flux, Φ, thus equals:

$$\text{nuclear reaction rate} = \sigma \Phi n \ ,$$

where n is the number of target nuclei.

Cross sections for individual types of nuclear reactions are marked with a corresponding subscript. The following standard notation is used for the most common neutron reactions:

- σ_s – neutron scattering cross section
- σ_a – neutron absorption cross section
- σ_c – capture cross section
- σ_γ – radiative capture cross section
- σ_f – fission cross section

Some of these cross sections are related to each other. A neutron absorbed by a nucleus causes either a capture reaction (cross section σ_c) or fission (σ_f). The absorption cross section is thus the sum of the capture and fission cross sections:

$$\sigma_a = \sigma_c + \sigma_f \ .$$

Non-fissile nuclei cannot undergo a fission reaction, which means their fission cross section equals zero. Their absorption cross section is equal to their neutron capture cross section.

5.4 Important reactions in nuclear technology

Reactions in reactor fuel

The most important nuclear reaction in fuel is of course fission. This is why we are going to discuss it in a special chapter. Apart from fission, another important reaction in fuel is the formation of plutonium. Plutonium is a transuranium element which does not exist in nature. It forms in a neutron capture reaction from the isotope ^{238}U which constitutes more than 95% of the uranium in the fuel of nuclear power plants. Radiative neutron capture in the isotope ^{238}U forms the isotope ^{239}U:

$$^{238}\text{U (n, }\gamma\text{) }^{239}\text{U .}$$

The newly formed and unstable ^{239}U nucleus undergoes two consecutive β^- decays, converting into the nuclide ^{239}Pu:

$$^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^- \qquad (t_{1/2} = 23.5 \text{ min}),$$
$$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^- \qquad (t_{1/2} = 2.4 \text{ days}).$$

Like ^{235}U, plutonium-239 is fissile and therefore highly important in nuclear technology. The described reaction thus results in new nuclear fuel. Neutron capture is also possible in plutonium, producing even heavier nuclei or higher elements.

Reactions in the reactor coolant and moderator

To regulate reactivity or the number of neutrons in the core of pressurized water reactors, boric acid, H_3BO_3 is dissolved in the coolant. Boron contains approximately 20% isotope ^{10}B, which is a strong absorber of thermal neutrons. The reaction is as follows:

$$^{10}\text{B (n, }\alpha\text{) }^{7}\text{Li .}$$

This is also the most frequent reaction used to detect slow or thermal neutrons. Fast neutrons can also trigger the following reaction in boron:

$$^{10}\text{B (n, 2}\alpha\text{) }^{3}\text{H .}$$

The resulting tritium is an important and undesirable source of radioactivity in the primary system. A minor source of tritium is also neutron reaction on lithium which is present in the coolant in the form of lithium hydroxide, LiOH, added to the coolant to regulate its pH. This lithium reaction is:

$$^{6}\text{Li (n, }\alpha\text{) }^{3}\text{H .}$$

Thermal neutrons can also be detected by using this reaction.

Fast neutrons (with energy over 10 MeV) also react with oxygen in water (reactor coolant) in the following reaction:

$$^{16}O\ (n, p)\ ^{16}N\ .$$

The resulting radionuclide, nitrogen-16 with a half-life of 7 s, is the main source of primary coolant radiation. As it also emits highly penetrating gamma rays, their detection can be used to measure pipe leakage in the steam generator.

Reactions in other primary system materials

The primary system in a reactor is mostly constructed of steel and other metals. Erosion and corrosion processes cause tiny metallic particles to pass into the coolant. The coolant brings them into the reactor core where they undergo nuclear reactions in the strong neutron flux, resulting in the formation of radioactive nuclides which are called **corrosion products**. Examples of such reactions are:

$$^{58}_{26}Fe(n,\gamma)^{59}_{26}Fe,$$
$$^{59}_{27}Co(n,\gamma)^{60}_{27}Co,$$
$$^{60}_{28}Ni(n,p)\ ^{60}_{27}Co,$$
$$^{58}_{28}Ni(n,p)^{58}_{27}Co,$$
$$^{50}_{24}Cr(n,\gamma)^{51}_{24}Cr.$$

Neutron sources

Every reactor needs a **neutron source**. The source provides a sufficient number of neutrons to be detected by the most sensitive neutron instrumentation even during reactor shutdown or start-up. The following nuclear reaction is often used for the neutron source:

$$^{9}Be\ (\alpha, n)\ ^{12}C\ .$$

This neutron source is made by mixing and compressing beryllium powder and an alpha emitter in a capsule. The alpha particle source can be e.g. radium-226 or americium-241.

5.5 Exercises

1. Complete the nuclear reactions:

$$^{63}Cu\ (n, \alpha)\ ....$$
$$^{9}Be\ (\gamma, ...)\ 2\ \alpha$$
$$^{6}Li\ (...., \alpha)\ ^{3}H$$
$$....\ (n, p)\ ^{60}Co$$

2. A copper-63 target is irradiated with deuterons. In some cases we obtain the nuclide ^{63}Zn, in others the nuclide ^{64}Zn, in others ^{64}Cu

and in yet others the nuclide ^{61}Ni. Write the nuclear reactions for these four cases.

3. Complete the reaction ^{3}He (n, ...) ^{3}H.

6 NUCLEAR FISSION

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Describe the fission mechanism.*
2. *Define fissile and fertile nuclides.*
3. *Illustrate the formation of ^{239}Pu.*
4. *Name the products of fission.*
5. *Sketch the dependence of fission fragment yield on mass number A.*
6. *Explain why fission fragments are unstable.*
7. *Explain the terms prompt and delayed neutrons.*
8. *Describe the distribution of energy released by fission.*
9. *Estimate the consumption of fissile material in a reactor operating at a certain power level.*
10. *Sketch the time course of decay heat.*

6.1 Spontaneous and induced fission, the fission mechanism

Nuclear fission is a process in which heavy nuclei ($A \geq 230$) *split* into two lighter nuclei due to the electrostatic repulsion between numerous protons. This also involves the release of two or three neutrons.

> Some very heavy transuranium nuclei can split of their own accord, but this is a very rare process. This phenomenon is called **spontaneous fission** and is grouped with rare types of radioactive decay. In californium-252, for example, 3% of decays are spontaneous fission (others are alpha decay). Since spontaneous fission also releases neutrons, nuclides with a large enough fraction of decay by spontaneous fission can serve as a source of neutrons. In old fuel elements subsequent neutron capture reactions result in formation of the curium isotopes ^{242}Cm and ^{244}Cm, which likewise have a relatively large fraction of spontaneous fissions.

The most common type of fission is **induced fission**, which is the consequence of an increase in the internal energy of a nucleus when it absorbs a particle, most frequently a neutron. This absorption produces an excited intermediate nucleus and its excitation makes it oscillate and deform. If the excitation energy is large enough, the intermediate nucleus may take on a shape resembling a dumb-bell. In such cases the electric repulsive forces between the two spheres forming the dumb-bell may overcome nuclear attraction and the nucleus splits. This process is shown in Figure 6.1.

Nuclides that undergo fission as soon as they capture a slow or thermal neutron are called **fissile nuclides**. Fissile nuclides can also split on capturing a fast neutron, but the likelihood of fission decreases with the increase in the kinetic energy of the neutron. The key fissile nuclides are ^{235}U and ^{239}Pu. The only natural fissile nuclide is ^{235}U.

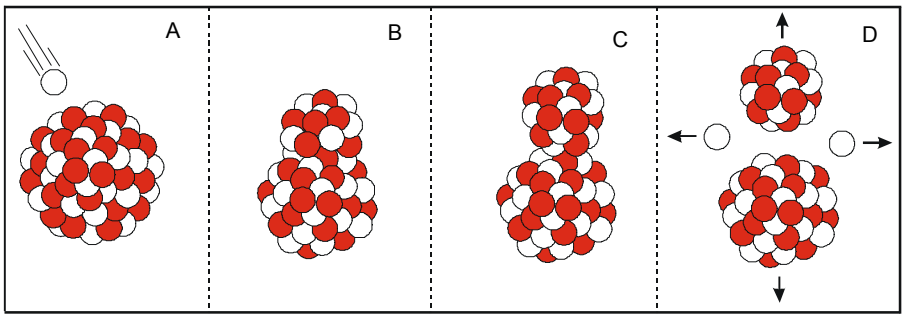


Figure 6.1: Nuclear fission.

Plutonium is formed from the uranium isotope ^{238}U via neutron capture followed by two beta minus decays:

$$^{238}_{92}\mathrm{U(n,\gamma)}\,^{239}_{92}\mathrm{U}\xrightarrow{\beta^-\,(t_{1/2}=23.5\ \mathrm{min})}\,^{239}_{93}\mathrm{Np}\xrightarrow{\beta^-\,(t_{1/2}=2.4\ \mathrm{days})}\,^{239}_{94}\mathrm{Pu}\ .$$

Since ^{238}U produces a fissile nuclide it is called a **fertile nuclide**. Fertile nuclides can only be split by fast neutrons.

A chain fission reaction can sustain itself only in the presence of a sufficient quantity of fissile nuclides, though a (smaller) share in the process is also contributed by fission by fast neutrons on fertile nuclides.

6.2 Products of fission

In fission a heavy nucleus splits into two medium-heavy nuclei called fission fragments and a few neutrons. Apart from these, each fission also gives off some gamma rays. Nuclei can split in hundreds of different ways; an example of one of the possible fissions is given in Fig. 6.2.

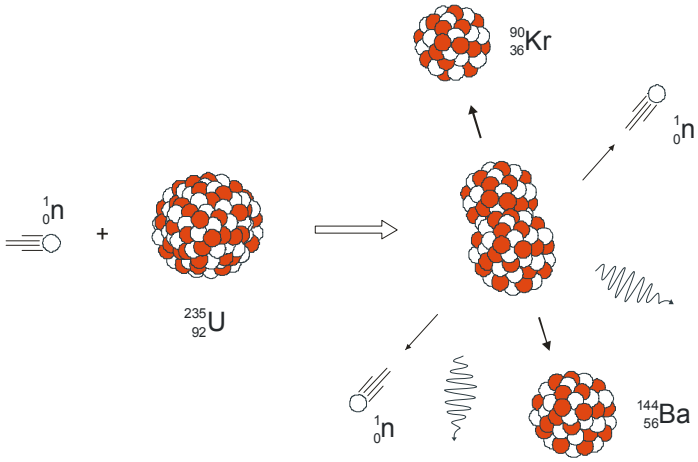


Figure 6.2: An example of products of fission.

To sum up, fission produces:

- fission fragments,
- prompt neutrons,
- prompt gamma rays,
- the release of a large amount of energy which is distributed as kinetic energy among the above listed products of fission.

Fission fragments are two medium-heavy nuclei which fly apart after fission with great kinetic energy (around 168 MeV). Very strong forces act on them as they move through matter, so they are stopped at a very short distance (a few 10 μm) in the fuel and their energy converted into internal energy of the fuel which increases its temperature.

A nucleus can split in a great number of ways, producing different fission fragments. Their mass numbers range from 75 to 160 and their atomic numbers from 30 to 64. Very rarely, a nucleus will split into two equal parts. It is more likely to split into a lighter and a heavier fragment. Fission fragments most frequently have a mass number of around 95 or 140. The **fission yield** is defined as the probability of producing a fission fragment with a certain mass number. Figure 6.3 shows the distribution of fission yields as a function of mass number, *A*.

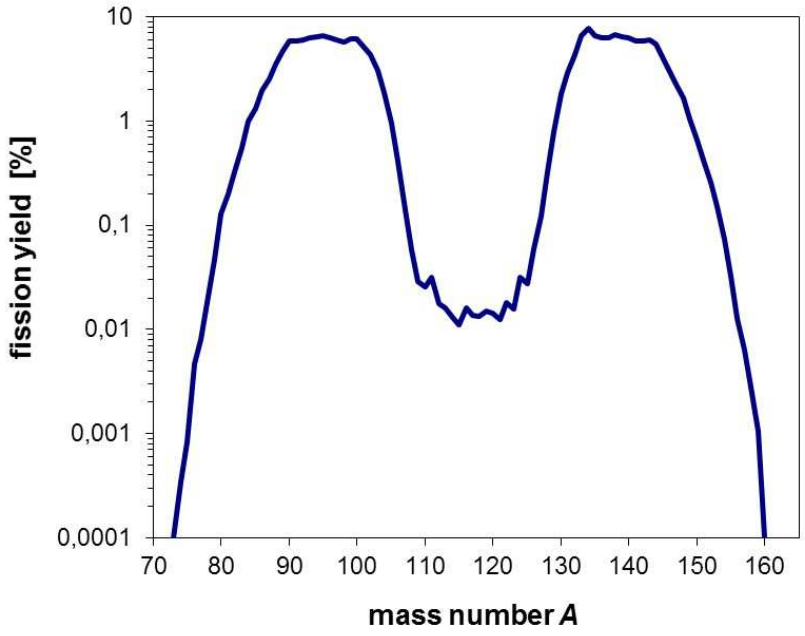


Figure 6.3: Fission yield of ^{235}U for fission with thermal neutrons.

In fission, the neutrons and protons are shared out among the fission fragments in roughly the same way as distributed in the initial nucleus. In medium-heavy fragments, the *N*/*Z* ratio is practically the same as in the initial heavy nucleus prior to fission (Figure 6.4). Stable medium-heavy nuclei have a smaller *N*/*Z* ratio than fission fragments. This means the fragments have an excess of neutrons, making them unstable and causing them to undergo β^- decay.

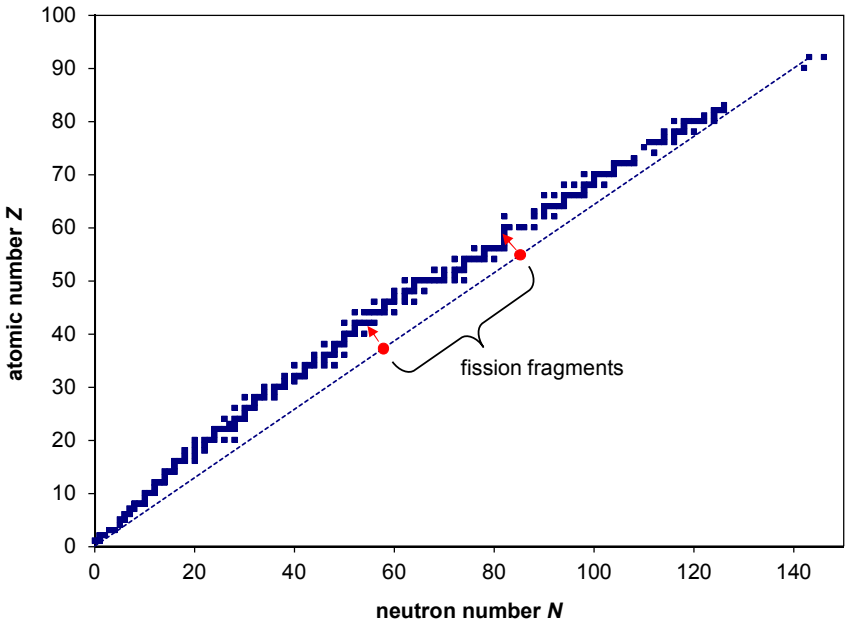


Figure 6.4: Table of nuclides with fission fragments.

Most fission fragments undergo 3 to 4 β^- decays before they turn into stable nuclei. Beta decays are typically combined with gamma decays. Fission fragments and their daughter nuclei are collectively called **fission products**. Around 300 different radioactive fission products are produced in total. The decay of fission products is very important for reactor operation for the following reasons:

- decays generate a large amount of energy which is also released as **decay heat** after reactor shutdown and has to be removed;
- radiation from decay is **dangerous to health** and since γ radiation in particular is highly penetrating, handling spent nuclear fuel is a very demanding task;
- the decay chains of certain fission fragments may involve emission of neutrons (**delayed neutrons**):
- some fission products have a significant impact on the course of the chain reaction because they are strong **neutron absorbers**.

Prompt neutrons are neutrons released directly by fission. The number of neutrons released varies from fission to fission; usually there are two or three. **The average number of neutrons per fission** is denoted by the parameter **ν**. Parameter ν depends on the type of nuclide undergoing fission and on the energy of the neutron causing fission. For ^{235}U fission with thermal neutrons, $\nu = 2.43$. The average kinetic energy of prompt neutrons is around 2 MeV, so in total they carry away approximately 5 MeV.

Besides fission fragments and prompt neutrons, fission is also directly followed by release of **prompt gamma rays**. On average, around 8 gamma rays are released per fission, in total carrying away around 7 MeV energy.

Some fission fragments also emit a neutron after β^- decay. This type of radioactive decay is called β-n decay and the neutron which is not produced until after such radioactive decay is a **delayed neutron**. The fission fragment that emits this neutron is called a delayed neutron precursor (Figure 6.5). The delay time of a delayed neutron is determined by the lifetime of its precursor. Such a precursor is e.g. ^{87}Br ($t_{1/2}$ = 56 s or τ = 80 s). There are at least 45 precursors of delayed neutrons.

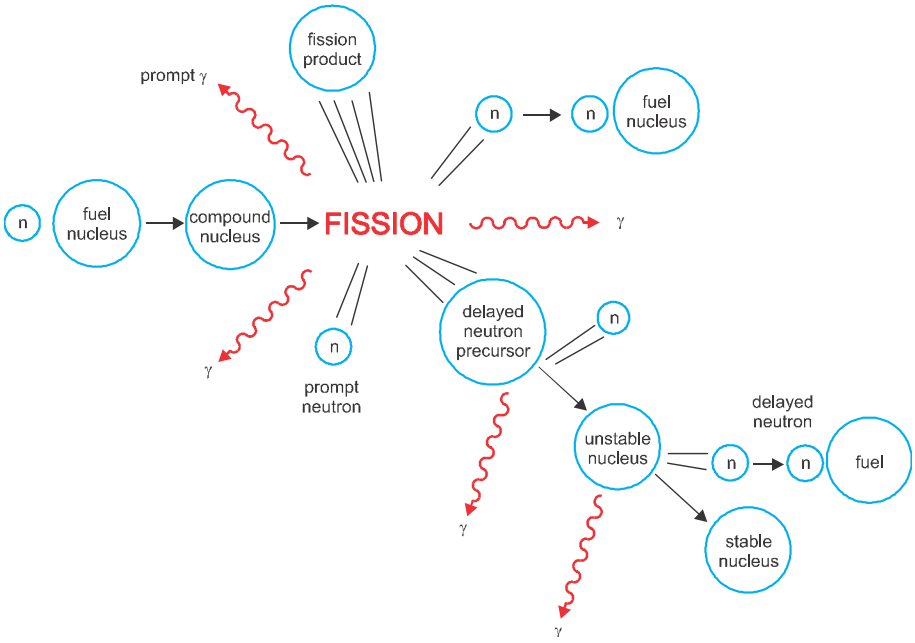


Figure 6.5: Production of delayed neutrons.

Since delayed neutron precursors have different half-lives and different fission yields, delayed neutrons are described by two parameters which express the average across all delayed neutron precursors:

- The **average lifetime** of delayed neutrons, τ, is the average delay time from fission to the release of delayed neutrons.
- The delayed neutron **fraction**, β, is the fraction of delayed neutrons in the total number of all the neutrons produced per fission.

What is particularly important is the delayed neutron fraction, β, which, though smaller than 1% for most nuclides, is crucial for successful reactor control.

Table 6.1 gives the most important parameters of delayed neutrons for specific nuclides and neutron energies.

Table 6.1: Total number of neutrons, ν, delayed neutron fraction, β, and the lifetime of delayed neutrons, τ, for different fission reactions.

Nuclide	Neutron energy	ν	β	τ [s]
^{235}U	thermal	2.43	0.0065	13
^{239}Pu	E = 0.025 eV	2.88	0.0022	15.4
^{235}U	fast	2.65	0.0064	12.8
^{238}U	E = 2 MeV	2.80	0.0164	7.7
^{239}Pu		3.15	0.002	14.6

It is evident from Table 6.1 that β varies from nuclide to nuclide, as well as relative to the energy of the neutrons causing fission. Fuel usually contains several different fissile nuclides and a reactor contains neutrons of varying energies, so we also define the **average delayed neutron fraction**, $\bar{\beta}$. Parameter $\bar{\beta}$ decreases with fuel burn-up, since fresh fuel contains more ^{235}U, which is burned up, and at the same time, ^{238}U turns into ^{239}Pu (which has a lower β than ^{235}U). A smaller share of the value of $\bar{\beta}$ is also contributed by ^{238}U fissions caused by fast neutrons. Thus, at the beginning of life or BOL, the typical value of $\bar{\beta}$ = 0.0071 and at the end of life or EOL it is reduced to $\bar{\beta}$ = 0.0054.

6.3 Fission energy balance

When listing the products of fission we also referred to the energy possessed by specific resulting particles: around 165 MeV by fission fragments, around 5 MeV by prompt neutrons and around 7 MeV by gamma rays. Not all neutrons cause fission; some are captured by nuclei in the core, chiefly by means of (n, γ) reactions (e.g. in ^{238}U). Gamma rays from such reactions contribute altogether around 6 MeV of energy. All of the energy mentioned so far is converted into heat in a very short time after fission through the interaction of particles with matter.

Table 6.2: Distribution of energy released by ^{235}U fission.

Prompt energy release		**range**
fission fragments	168 MeV	~ 10 μm
prompt neutrons	5 MeV	0.1 – 1 m
prompt gamma rays	7 MeV	0.1 – 1 m
gamma rays from (n, γ) reactions	6 MeV	0.1 – 1 m
total prompt energy release	**186 MeV**	
Delayed energy release		
β from fission fragment decay	7 MeV	~ mm
γ from fission fragment decay	6 MeV	0.1 – 1 m
β and γ from nuclei produced by (n, γ)	1 MeV	0.001 – 1 m
total delayed energy release	**14 MeV**	
total energy release	**200 MeV**	

Some energy is also released with a certain time delay relative to fission – this is mostly energy released by β^- and γ decays of fission products and radioactive nuclides resulting from (n, γ) reactions. The distribution of such energy with the products of fission, together with the estimated distance at which the energy is released, is given in Table 6.2.

The total energy released by the fission of one ^{235}U nucleus is thus 200 MeV. Based on this piece of information we can calculate how many nuclei are fissioned in a reactor operating for1 day at 1 MW power, or how many fissions are required to release 1 MWd energy:

$$1\ \text{MWd} = \frac{1\ \text{MWd}}{200\ \text{MeV/fission}} = \frac{10^6\,\frac{\text{J}}{\text{s}} \cdot 24 \cdot 3600\ \text{s}}{200 \cdot 10^6 \cdot 1.6 \cdot 10^{-19}\,\frac{\text{J}}{\text{fission}}}$$
$$= 2.7 \cdot 10^{21}\ \text{fissions.}$$

As shown, 1 MWd energy is released by the fission of $2.7 \cdot 10^{21}$ atoms of ^{235}U. Each of these atoms has a mass of 235 u (atomic mass units). The total mass of ^{235}U required to release 1 MWD energy is therefore equal to:

$$m = 2.7 \cdot 10^{21} \cdot 235 \cdot 1.66 \cdot 10^{-27}\ \text{kg} = 1.05 \cdot 10^{-3}\ \text{kg} \approx 1\ \text{g}\ .$$

One MWd energy is generated from the fission of approximately 1 g of ^{235}U, which roughly applies also to the other fissile nuclides. Hence we may generalize:

$$1\ \text{MWd} \approx 1\ \text{g fissile nuclides}\ .$$

Example:
A reactor operates for 30 days at a (thermal) power of 3000 MW. What is the mass of the fissioned fuel?
Answer:

3000 MW × 30 d = 90000 MWd ≈ 90 kg.

6.4 Decay heat

When discussing the distribution of released fission energy we learned that approximately 14 MeV of the total 200 MW of energy is released with a certain delay relative to fission. This delay is particularly important after reactor shutdown; the chain reaction is interrupted but energy – in the form of heat – continues to be released. This energy is called residual heat or decay heat, because it results from the radioactive decay of fission products.

Decay heat is defined as the energy released in the core after reactor shutdown. Being proportional to the reactor's operating power before

shutdown, it is expressed as a fraction of this power. Immediately after shutdown, decay heat rapidly diminishes with time due to the rapid decay of short-lived fission products. The downward trend then slows down due to the slower decay of long-lived nuclides. The decay heat also depends on the length of operation prior to shutdown. The longer a reactor operates, the more long-lived nuclides accumulate in the core and the slower the decay heat diminishes.

Figure 6.6 shows the time course of decay heat after reactor shutdown for various times of reactor operation prior to shutdown.

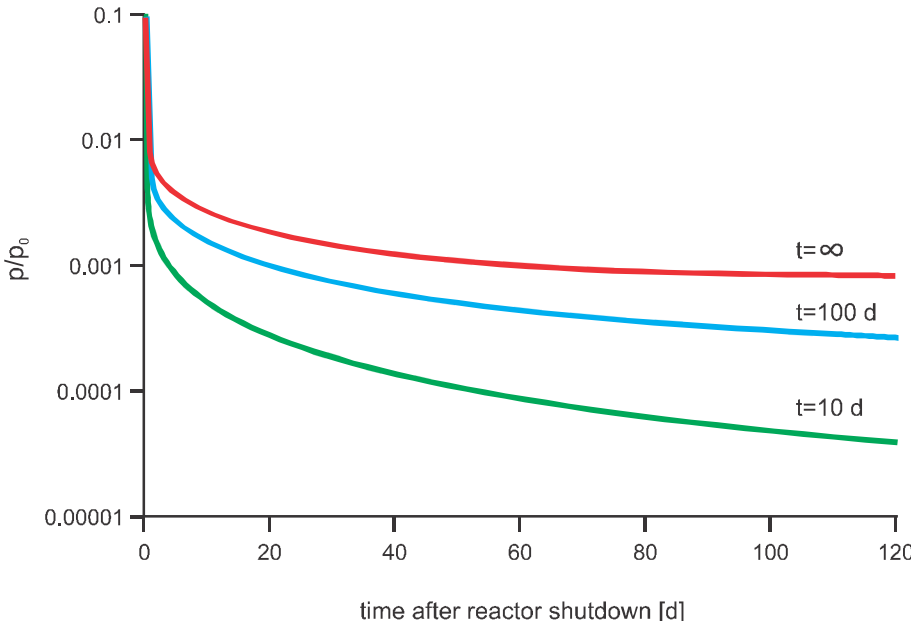


Figure 6.6: Time dependence of the fractional decay heat for various reactor operating times prior to shutdown.

Table 6.3 gives the decay heat for a 3000 MW_{th} reactor after a longer period of operating at full power.

Table 6.3: Decay heat after a long period of operation at 3000 MW_{th}.

Time after shutdown	Fraction of full power	Decay heat
1 s	6.2%	185 MW
1 min	3.6%	107 MW
1 hour	1.3%	38 MW
8 hours	0.6%	19 MW
1 day	0.4%	13 MW
1 week	0.2%	7 MW
1 month	0.1%	4 MW

Due to decay heat, the core must continue to be cooled even after

shutting down the reactor, i.e. after stopping the fission chain reaction. Decay heat needs to be removed continuously. Failing this, the temperature of the fuel would rise and lead to fuel damage or possibly even fuel melting. This is why nuclear power plants have a special system for removing decay heat. Decay heat is also released in the spent fuel pit (Figure 6.7) which likewise has its own decay heat removal system. Decay heat represents one of the most serious nuclear safety issues.

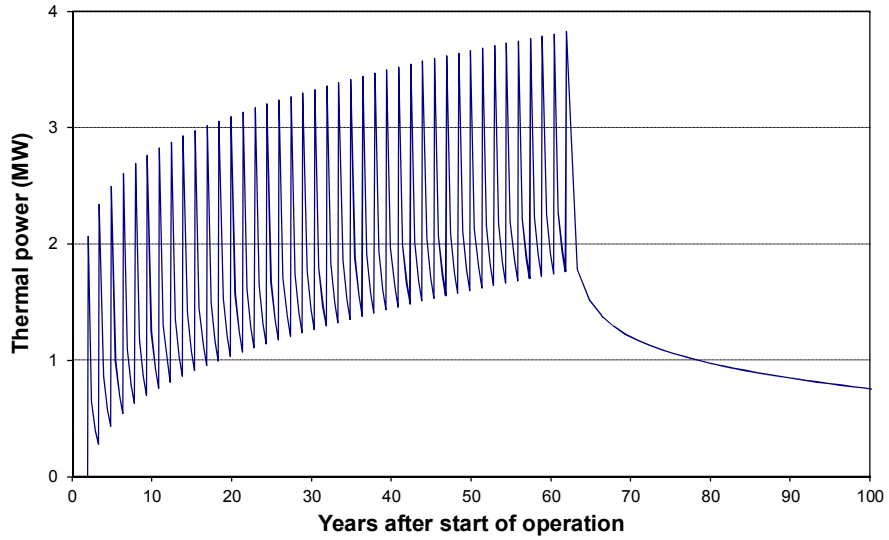


Figure 6.7: Decay heat in the spent fuel pit of a 3000 MW_{th} reactor, assuming 18-month cycle and 60 years of operation.

6.5 Exercises

1. Complete the table which relates to the general notation for the fission reaction $^{235}U + n \rightarrow X + Y + b\ n$:

X	Y	*b*
^{140}Xe	-	1
^{139}I	-	2
-	^{100}Zr	2
^{141}Cs	^{92}Rb	-

2. A research reactor using 20% enriched fuel has a thermal power of 250 kW. How much uranium-235 is fissioned in one day?
3. A nuclear reactor containing 10 tons of natural uranium operates at a thermal power of 50 MW. What percentage of uranium is burned up in one week of operation? Natural uranium contains 0.72% of uranium-235.

7 NEUTRON CYCLE

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Explain the slowing down of neutrons in the core.*
2. *Give the properties of an effective moderator.*
3. *Define individual factors in the neutron cycle.*
4. *Define the multiplication factor, k.*
5. *Explain a chain reaction.*
6. *Define reactivity and the corresponding units.*

A reactor core contains neutrons of various energies. In terms of energy, they can be roughly divided into three categories:

- fast neutrons with $E > 0.1$ MeV,
- epithermal neutrons with $1\ \text{eV} < E < 0.1$ MeV,
- thermal neutrons with $E < 1$ eV.

7.1 Slowing-down of neutrons in the reactor core

When produced by fission, neutrons have an average energy of about 2 MeV. They are slowed down by scattering on nuclei in the substance through which they move. This lowers their kinetic energy until it becomes comparable to the kinetic energy of thermally moving nuclei in matter. Such neutrons are called thermal neutrons and the process in which they originate is called thermalization. Neutrons are most effectively slowed down by elastic scattering on light nuclei. In light-water reactors, this involves water or hydrogen.

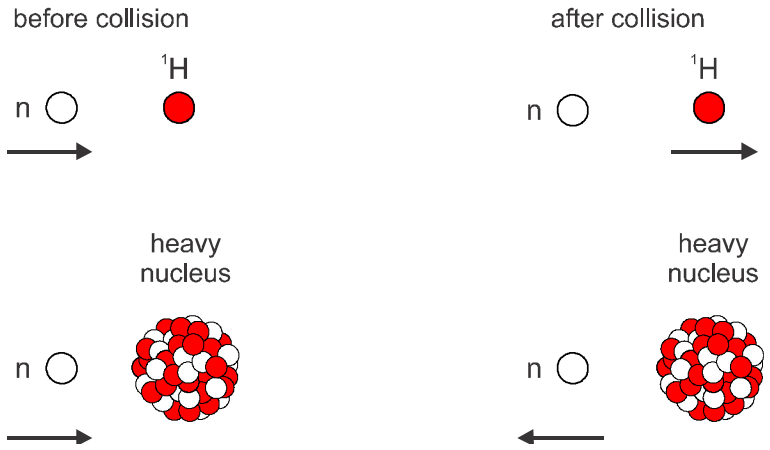


Figure 7.1: Illustration of elastic neutron scattering on a light and a heavy nucleus.

The substance in the reactor core which slows down fast neutrons is called the **moderator**. A moderator is effective if:

- a neutron loses a lot of energy in a single collision,
- it has a large macroscopic cross section for neutron scattering,

- it has a small macroscopic cross section for neutron absorption.

It is also important that the moderator does not activate, is a good heat conductor and is cheap.

Neutrons created by fission are fast neutrons. However, it is thermal neutrons which induce further fissions or maintain the chain reaction most efficiently. To better understand the operation of a reactor, we will take a closer look at the sequence of major events that neutrons undergo from their "birth" to their "death" by absorption in some material or leakage from the reactor core. This process is called the **neutron cycle**. Let us assume that, at a certain moment, we have n fast neutrons which were born by fission with thermal neutrons.

7.2 Fast fission factor ε

As soon as they are produced, some of the fast neutrons cause fast fission, mainly of ^{238}U.

The fast fission factor ε is defined as:

$$\varepsilon = \frac{\text{number of fast neutrons produced by fission with neutrons of all energies}}{\text{number of fast neutrons produced by fission with thermal neutrons}}$$

If, at the beginning, we had n fast neutrons produced by thermal fission, the fast fissions leave us with $\varepsilon \cdot n$ fast neutrons.

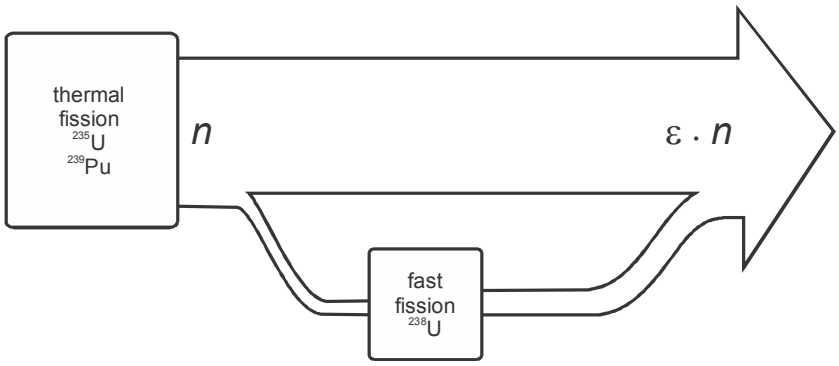


Figure 7.2: Fast fission factor ε.

7.3 Fast non-leakage factor P_f

Some of the fast neutrons escape from the core. The **fast non-leakage factor**, P_f, is the probability that a fast neutron will not escape from the core. It is defined as:

$$P_f = \frac{\text{number of fast neutrons which remain in the core}}{\text{number of fast neutrons produced by fission with neutrons of all energies}}$$

If we have $\varepsilon \cdot n$ fast neutrons at the beginning, $P_f \cdot \varepsilon \cdot n$ will remain in

the core.

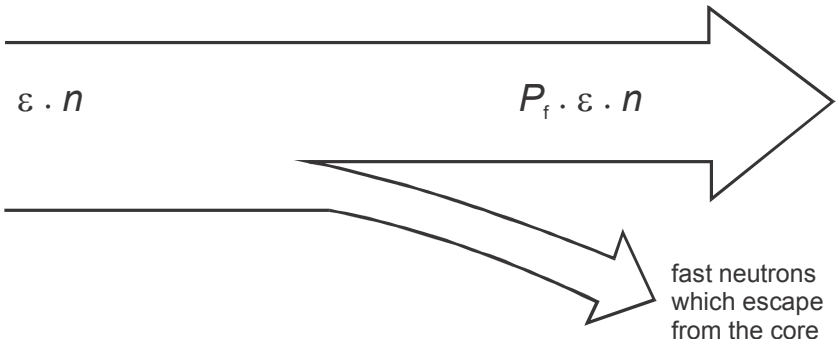


Figure 7.3: Fast non-leakage factor, P_f.

7.4 Resonance escape probability *p*

The fast neutrons which remain in the core slow down, i.e. their energy decreases. As they slow down, some of the neutrons are absorbed (captured). The highest probability for neutron absorption is in the epithermal energy range. In this range, some nuclei have very large absorption cross sections (resonances) at certain energies. Neutron absorption in this range is called resonance absorption. Most resonance absorption occurs in ^{238}U nuclei and ^{240}Pu nuclei (the latter is produced during reactor operation). **Resonance escape probability, *p***, is the probability that fast neutrons which remain in the core and slow down will not be absorbed in resonances:

$$p = \frac{\text{number of fast neutrons slowed down to thermal energies}}{\text{number of fast neutrons that start slowing down}}$$

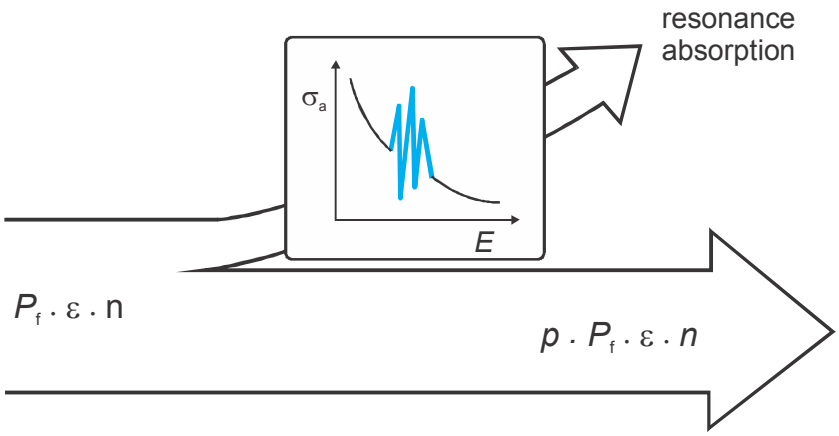


Figure 7.4: Resonance escape probability, *p*.

The number of fast neutrons which remain in the core and start slowing down is $P_f \cdot \varepsilon \cdot n$, and the number of neutrons which slow down to thermal energies is $p \cdot P_f \cdot \varepsilon \cdot n$.

7.5 Thermal non-leakage factor P_t

The neutrons which are not absorbed in resonances thermalize (slow down to thermal energies). Some of these neutrons escape from the core. The **thermal non-leakage factor**, P_t, is the probability that thermal neutrons will not escape from the core:

$$P_t = \frac{\text{number of thermal neutrons which remain in the core}}{\text{number of neutrons which slow down to thermal energies}}$$

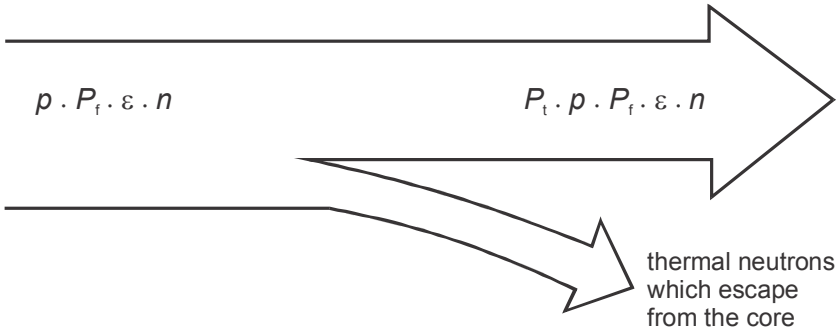


Figure 7.5: Thermal non-leakage factor, P_t.

The number of neutrons which thermalize is $p \cdot P_f \cdot \varepsilon \cdot n$, and the number of thermal neutrons which remain in the core is $P_t \cdot p \cdot P_f \cdot \varepsilon \cdot n$.

7.6 Thermal utilisation factor *f*

All of thermal neutrons which remain in the core end up being absorbed in one of the various materials in the core. They are absorbed in fuel, construction materials, water, the boron dissolved in water, the control rods if inside the core, etc. The **thermal utilisation factor**, ***f***, gives the fraction of thermal neutrons absorbed in fuel relative to absorption elsewhere in the core:

$$f = \frac{\text{number of thermal neutrons absorbed in the fuel}}{\text{number of thermal neutrons absorbed in all core materials}}$$

The number of thermal neutrons which remain in the core is $P_t \cdot p \cdot P_f \cdot \varepsilon \cdot n$, and the number of neutrons absorbed in fuel is $f \cdot P_t \cdot p \cdot P_f \cdot \varepsilon \cdot n$.

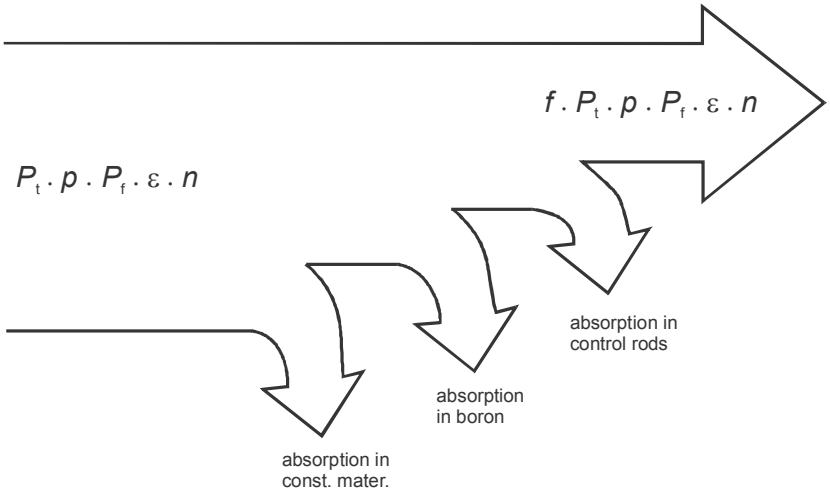


Figure 7.6: Thermal utilisation factor, f.

7.7 Neutron yield per absorption η

Not all the neutrons which are absorbed in fuel cause fission. The **neutron yield per absorption**, η, gives the number of fast neutrons which are released in thermal fission per thermal neutron absorbed in fuel:

$$\eta = \frac{\text{number of fast neutrons from thermal fission}}{\text{number of thermal neutrons absorbed in fuel}}$$

The number of thermal neutrons which are absorbed in fuel is

$$f \cdot P_t \cdot p \cdot P_f \cdot \varepsilon \cdot n,$$

and the number of fast neutrons produced in fission is

$$\eta \cdot f \cdot P_t \cdot p \cdot P_f \cdot \varepsilon \cdot n.$$

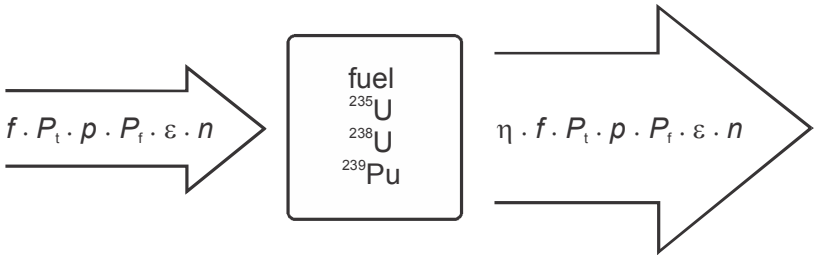


Figure 7.7: Neutron yield per absorption, η.

7.8 Multiplication factor k

The full neutron cycle is shown in the Figure 7.8.

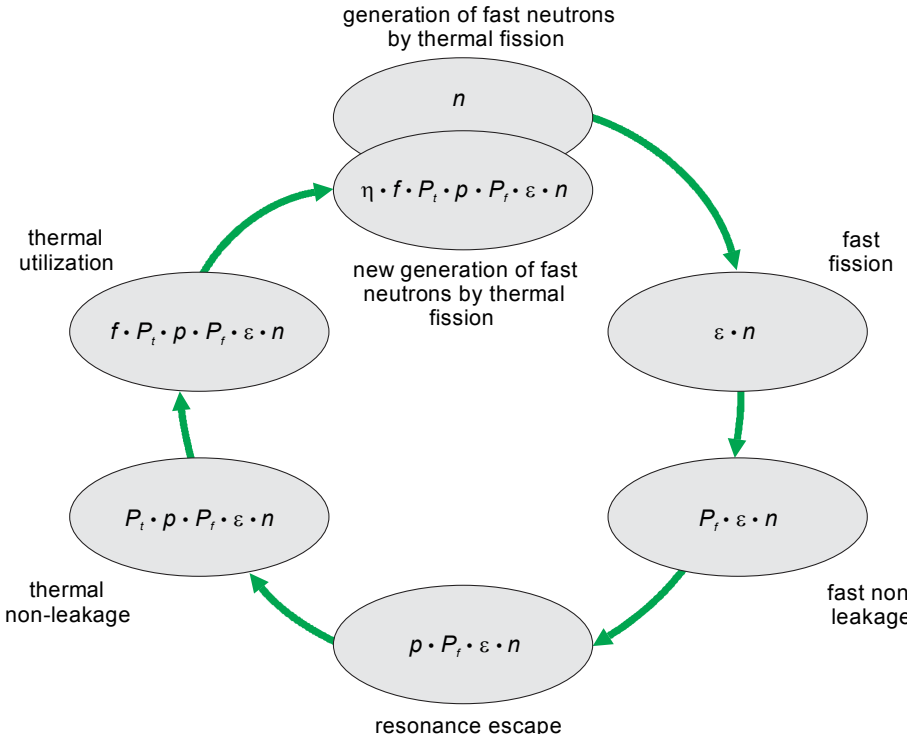


Figure 7.8: The neutron cycle.

At the beginning of the cycle we had a generation of n fast neutrons from thermal fission. At the end of the cycle, we gained a new generation of

$$\eta \cdot f \cdot P_{\mathrm{t}} \cdot p \cdot P_{\mathrm{f}} \cdot \varepsilon \cdot n$$

fast neutrons, also produced by thermal fission. We could say that after one cycle the neutrons multiplied by a factor of $\eta \cdot f \cdot P_{\mathrm{t}} \cdot p \cdot P_{\mathrm{f}} \cdot \varepsilon$.

The multiplication factor, k, is defined as:

$$k = \frac{\text{number of neutrons in a given generation}}{\text{number of neutrons in the previous generation}}$$

and expressed in the form of a **six-factor formula**:

$$k = \eta \cdot f \cdot P_{\mathrm{t}} \cdot p \cdot P_{\mathrm{f}} \cdot \varepsilon.$$

7.9 Chain reaction

Neutrons produced by fission spend some time in the core. Some escape from the core, some are captured and some cause new fissions. This produces new neutrons, some of which trigger further fissions,

etc. Each generation of neutrons leads to the birth of a new generation. This process is called a **chain reaction** (Fig. 7.9).

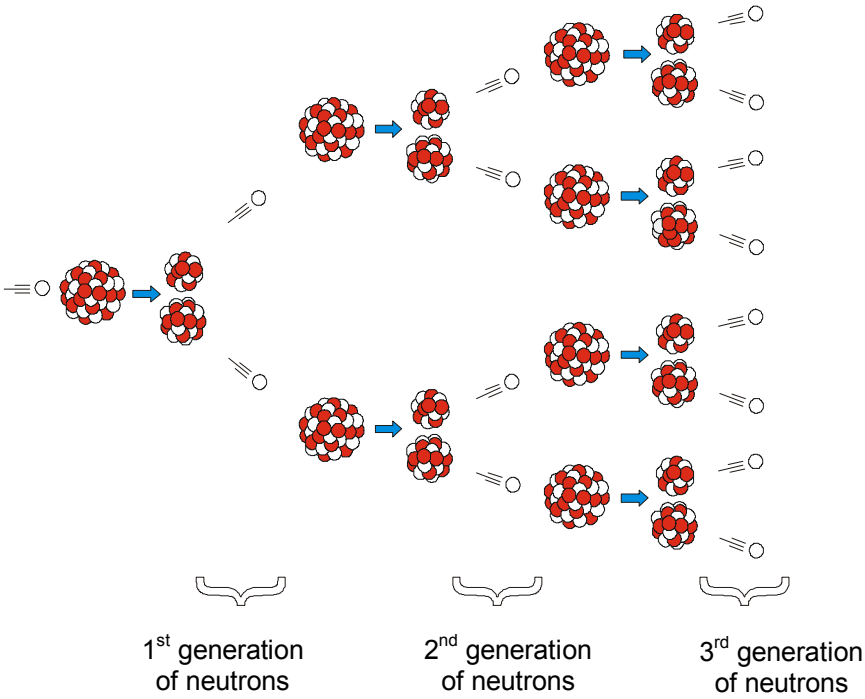


Figure 7.9: Chain reaction in the case $k = 2$.

- A chain reaction in which the number of neutrons in the reactor does not change with time is called a **critical chain reaction**. The number of fissions per unit time is constant, so reactor power is also constant. In such a case:

$$k = 1.$$

- In a **supercritical chain reaction** the number of neutrons and the number of fissions per unit time increase with time. Reactor power also increases with time. This is when:

$$k > 1.$$

- In a **subcritical chain reaction** the number of neutrons, the number of fissions per unit time, and of course the reactor power decrease with time. In this case:

$$k < 1.$$

7.10 Reactivity

We are usually interested in reactor conditions in which the value of the multiplication factor, k, approaches 1. To avoid using the multiplication factor, k, which has a large number of decimal places,

let us introduce a new quantity: reactivity, ρ. It is defined as:

$$\rho = \frac{k-1}{k}.$$

ρ represents the departure from criticality. The following applies:

$k < 1 \Rightarrow \rho < 0 \Rightarrow$ **reactor is subcritical,**
$k = 1 \Rightarrow \rho = 0 \Rightarrow$ **reactor is critical,**
$k > 1 \Rightarrow \rho > 0 \Rightarrow$ **reactor is supercritical.**

Like the multiplication factor, reactivity is a dimensionless quantity. For practical reasons it is supplemented with units which suggest how it was calculated. Several notations are used:

1. **Δk/k**: reactivity is calculated and the number is followed by Δk/k,

2. **% Δk/k**: 1% Δk/k = 0.01 Δk/k,

3. **pcm**[1]: 1 pcm = 0.00001 Δk/k,

4. **$**: 1 $ = $\bar{\beta}$ Δk/k.

It is desirable for a nuclear power plant to operate at constant power. This occurs when the reactor is critical, the multiplication factor $k = 1$ and the core reactivity $\rho = 0$. However, core properties modify with operation and in turn change the multiplication factor, k, or reactivity, ρ. These properties can also be influenced by the operator by manipulating the core. This is achieved in particular by adding or removing boron from the reactor coolant or by withdrawal or insertion of the control rods in the core. If the change reduces core reactivity, we say that **negative reactivity** is added to the core. The effect of reducing reactivity is equivalent to adding the negative value of reactivity to the initial reactivity. On the other hand, if influences exerted on the core increase its reactivity, we say that **positive reactivity** is added. If the operators want to increase reactor power, they must add enough positive reactivity by withdrawing the control rods to make the reactor supercritical and thereby increase power. Power is decreased in a similar way; negative core reactivity has to be ensured and power will start to diminish.

7.11 Questions

1. What are the properties of a good moderator?
2. Define the multiplication factor, *k*.
3. How is the fast fission factor, ε, defined?

[1] "pcm" stands for "per cent mille"

4. How is the fast non-leakage factor, P_f, defined?
5. How is the resonance escape probability, p, defined?
6. How is the thermal non-leakage factor, P_t, defined?
7. How is the thermal utilisation factor, f, defined?
8. How is neutron yield per absorption, η, defined?
9. What reactivity, ρ, in pcm units corresponds to a reactivity of 0.1% $\Delta k/k$?

8 REACTOR KINETICS

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Explain the exponential increase in reactor power.*
2. *Define the period T.*
3. *Define the start-up rate or SUR.*
4. *Describe the relationship between the multiplication factor, reactivity, the period and start-up rate and sketch how power changes for different k values.*
5. *Explain a chain reaction by taking and without taking delayed neutrons into account.*
6. *Define prompt criticality.*
7. *Sketch the reactor power response to a step change in reactivity.*

During reactor operation at sufficiently low power levels, thermal power is so low that the temperature of fuel and coolant and hence the properties of the core do not change perceptibly. Any change in power is caused only by external factors, for example by withdrawal or insertion of the control rods in the core. We say that in this operating range there are no temperature feedback effects.

There are three types of excore neutron detectors in a typical PWR: BF_3 proportional detectors are used for the shutdown condition, compensated ionization chambers for the low and operating power range, and ionization chambers for the operating power range. The entire power range of a power reactor comprises approximately 12 decades of neutron flux density. Fig. 8.1 illustrates the power ranges covered by the respective detectors.

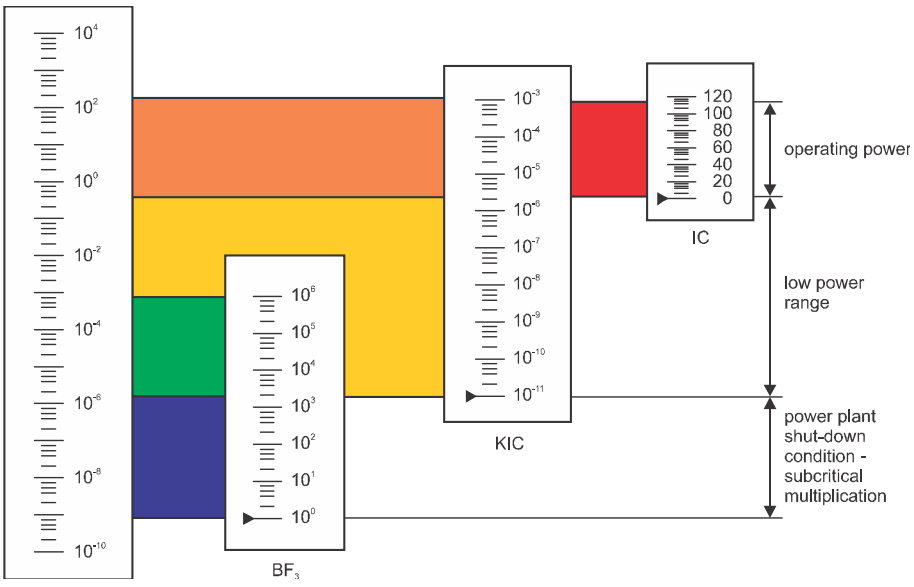


Figure 8.1: Measurement ranges for neutron flux density.

8.1 Time dependence of reactor power

Let us say that l is the average lifetime of neutrons. This is the time from the moment neutrons are born to the moment they disappear. At a certain point we have N neutrons in the core. When a single neutron cycle, which lasts for time l, has passed, we get $k \cdot N$ new neutrons. The difference in neutrons from cycle start to finish is written as:

$$\Delta N = k \cdot N - N = (k - 1) \cdot N = \Delta k \cdot N,$$

or the change in the number of neutrons per unit time:

$$\frac{\Delta N}{\Delta t} = \frac{\Delta k \ N}{l}.$$

The solution to this equation is an exponential function:

$$N = N_0 \cdot e^{\frac{\Delta k}{l} \cdot t},$$

where N_0 is the number of neutrons at time $t = 0$ and N is the number of neutrons at time t.

By replacing $l/\Delta k$ with T we get:

$$N = N_0 \cdot e^{\frac{t}{T}}.$$

Since power is proportional to the neutron population in the reactor, the same equation applies to power:

$$P = P_0 \cdot e^{\frac{t}{T}}.$$

T is called the **reactor period**. It is measured in seconds.

The reactor period is the time in which reactor power changes by factor *e*.

The equation generally applies to an interval of power change after the initial transient has passed. Period ***T*** calculated from the equation ***l/Δk*** is an approximation and applies when dealing with very small reactivities.

The rate at which power changes is usually not expressed by the reactor period T, but by the **start-up rate, SUR**, defined by the equation:

$$P = P_0 \cdot 10^{SUR \cdot t},$$

where P_0 is the initial reactor power and P is the reactor power at the

end of time t, given in minutes. SUR is expressed in min^{-1} and tells us how many decades a minute the reactor power changes.

The relationship between SUR and the reactor period is expressed by the equation:

$$SUR = \frac{26}{T}.$$

The value of the period, T, should be entered in seconds to get the value for SUR in decades per minute.

Supercritical reactor

The following applies to a supercritical reactor:

$$k > 1 \Rightarrow \rho > 0 \Rightarrow T > 0 \Rightarrow SUR > 0.$$

Reactor power increases exponentially with time. How quickly the power changes depends on the size of $\boldsymbol{k}$, or $\boldsymbol{\Delta k}$, or reactivity $\boldsymbol{\rho}$ The higher $\boldsymbol{k}$, the higher $\boldsymbol{\Delta k}$, and the higher the core reactivity $\boldsymbol{\rho}$. The higher $\boldsymbol{\rho}$, the faster reactor power increases and the higher is $\boldsymbol{SUR}$ or the shorter the reactor period $\boldsymbol{T}$. The increase in reactor power for different values of $\boldsymbol{k}$ is shown below (Fig. 8.2).

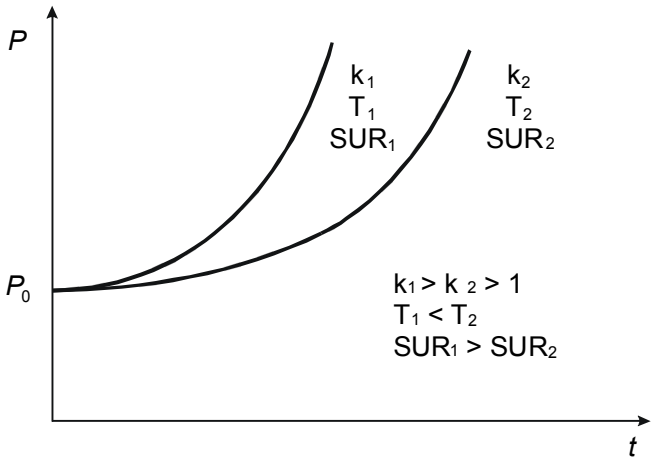


Figure 8.2: Supercritical reactor dynamics.

When increasing the power, the reactor is supercritical. This does not mean it is going to explode. The power increase is controlled by the operators or regulation systems, and safety is ensured by protection systems which automatically shut down the reactor in case of faulty operation. The operators must increase reactor power slowly and in accordance with the procedure which specifies the maximum start-up rate.

Critical reactor

The following applies to a critical reactor:

$$k = 1 \Rightarrow \rho = 0 \Rightarrow T = \infty \Rightarrow SUR = 0.$$

The reactor power is constant and does not change with time. During operation at constant power, $k = 1$ or $\rho = 0$. At higher power levels, the number of fissions per unit of time (or number of neutrons) is higher than at lower power levels, but constant in a critical reactor (it does not change with time). A critical reactor therefore does not present a risk but represents normal operation at constant power. Fig. 8.3 shows the time dependence of critical reactor power.

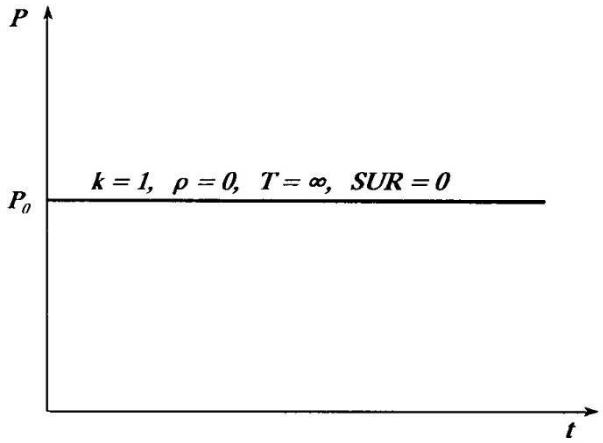


Figure 8.3: Critical reactor dynamics.

Subcritical reactor

The following applies to a subcritical reactor:

$$k < 1 \Rightarrow \rho < 0 \Rightarrow T < 0 \Rightarrow SUR < 0.$$

The reactor power falls exponentially with time. When decreasing power, the reactor is always subcritical. The smaller is k, the smaller the reactivity ρ (more negative), the faster (at a shorter negative period T or higher negative SUR) the reactor power decreases with time. Fig. 8.4 shows the decrease in reactor power for different values of the multiplication factor k.

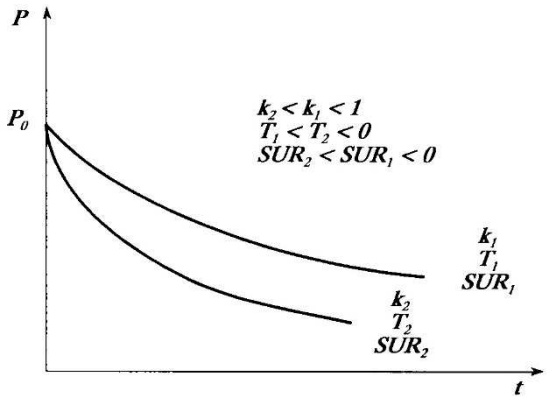


Figure 8.4: Subcritical reactor dynamics.

8.2 Neutron lifetime

This is the time from the birth to the disappearance of a neutron (escape, absorption). It is divided into:

- the birth time; the time from fission to neutron release,
- the slowing-down time; the average time from a neutron's release to its thermalization,
- the diffusion time; the average time from a neutron's thermalization to its disappearance. This is the time during which neutrons on average no longer lose any kinetic energy.

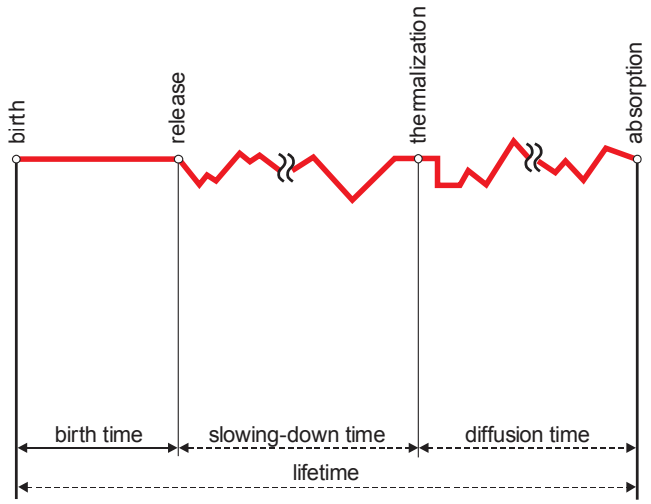


Figure 8.5: Neutron lifetime.

Chain reaction with prompt neutrons

The birth time of prompt neutrons is of the order of magnitude of 10^{-14} s. In a pressurized water reactor core, the slowing-down and diffusion times are of the order of magnitude of 10^{-5} s. The lifetime of prompt neutrons in a pressurized water reactor core is of the order of magnitude of 10^{-4} s.

We can see how a power increase takes place in the next example.

Example:
Let us suppose a reactor is operating at 10 kW constant power. This is the low power range where no perceptible temperature changes in the fuel and coolant occur due to power change. The operator withdraws the control rods approximately 10 steps which roughly corresponds to the reactivity ρ = 70 pcm. This reactivity corresponds to a multiplication factor $k \approx 1.0007$. The reactor is supercritical and power starts to increase at a period:

$$T = l/\Delta k = 10^{-4}\ \text{s}/0.0007 = 0.143\ \text{s}.$$

After one second, the power would be:

$$P = P_0\, e^{t/T} = P_0\, e^{1\text{s}/0.143\ \text{s}} \approx 1100\, P_0.$$

Thus, in one second, power would increase by a factor of 1100; from its initial value of 10 KW to 11 MW.

If we have a chain reaction with prompt neutrons only, we have a very short period (or high SUR) even though the reactivity of the core is small. Such fast changes in power increase are impossible to control.

Chain reaction with prompt and delayed neutrons

The slowing-down and diffusion times of delayed neutrons are comparable to those of prompt neutrons. Both are born as fast neutrons – the delayed neutrons with slightly lower energies, which are of the order of magnitude of a few 100 keV. The birth time of a delayed neutron is essentially the lifetime of its precursor, which has been denoted by τ. Since the lifetimes of delayed neutron precursors are much longer than the slowing-down or diffusion time, we can simplify by saying the lifetime of delayed neutrons is approximately equal to τ.

The average lifetime of all neutrons, $\bar{l}$, is defined as the average of prompt and delayed neutron lifetimes weighted by their respective fractions in the entire neutron population:

$$\bar{l} = (1 - \beta) \cdot l_p + \beta \cdot \tau \, .$$

l_p is used to denote the lifetime of prompt neutrons. The value obtained for the average lifetime of all neutrons in pressurized water reactors is thus:

$$\bar{l} = (1 - 0.0065) \cdot 10^{-4} \text{ s} + 0.0065 \cdot 13 \text{ s} \approx 0.1 \text{ s} \quad .$$

Example:
Let us perform the same power increase experiment as in the previous example.
We get a reactor period value of:

$$T = l/\Delta k = 0.1 \text{ s}/0.0007 = 143 \text{ s}.$$

In one second, the power would be:

$$P = P_0 \, e^{t/T} = P_0 \, e^{1\text{s}/143 \text{ s}} = P_0 \, e^{0.007} = 1.007 \, P_0,$$

and in 100 seconds:

$$P = P_0 \, e^{100 \text{ s}/143 \text{ s}} = 2.0 \, P_0.$$

In 1 s, the power increases by 0.7% and in 100 s it doubles. Such slow changes are well within the operator's control. Thus delayed neutrons, despite their relatively small fraction in the neutron population, fundamentally change how the reactor behaves.

The relationship between core reactivity and the reactor period is detailed in the figure below (Fig. 8.6), where reactivity ρ is given in units \$ as the abscissa and the reactor period $\boldsymbol{T}$ in seconds as the ordinate.

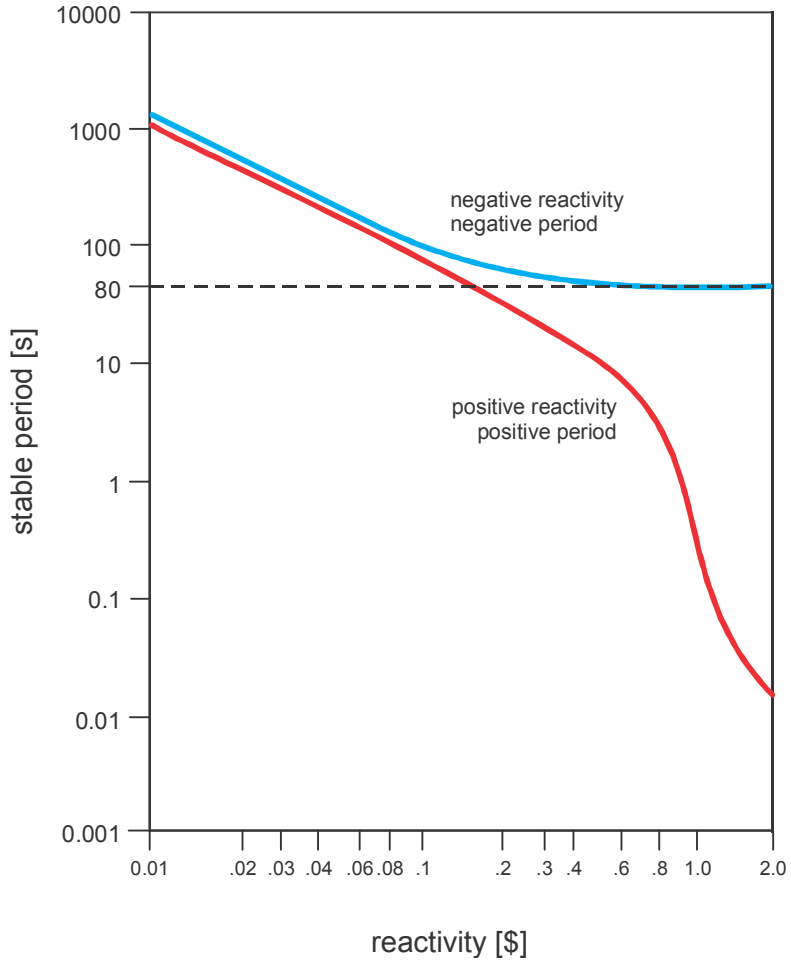


Figure 8.6: Dependence of reactor period on reactivity.

The upper curve represents the dependence between negative reactivity values and the corresponding negative period, and the bottom curve between positive reactivity values and the corresponding positive period. In case of negative reactivity, the result for period T is negative and in the case of positive reactivity it is positive. The figure reveals that the shortest negative period is -80 s. The reactor cannot be shut down faster than it takes for the most long-lived precursors of delayed neutrons to decay.

8.3 Prompt criticality

For reactivities larger than or equal to $\bar{\beta}$, i.e. 1$, the reactor period is determined by the lifetime of prompt neutrons. Hence it is very short. When positive reactivity is higher than $\rho = \bar{\beta}$, the reactor is said to be **prompt critical**.

To maintain a critical chain reaction the delayed neutrons are no longer needed. The period is determined by the lifetime of prompt neutrons and is very short.

Rule: reactor reactivity must **always be kept smaller than** $\bar{\beta}$ (1$) both during start-up and power increase.

8.4 Reactor power response to a step change in reactivity

Let us see how the reactor responds to a positive reactivity insertion if the reactor previously operated at constant low power without temperature feedback influences. The operator can produce this kind of change in reactivity by withdrawing the control rods a few steps from the core. This case is shown in the Figure 8.7. As soon as reactivity is inserted there is a power surge, a transient caused by a momentary increase in the fraction of prompt neutrons. After this transient, power rises at a stable period, *T*. Since there are no temperature feedback effects, power rises until the operator lowers the control rods into the core to balance out the initial inserted positive reactivity to zero. Since reactivity $\rho = 0$, power stabilizes and is higher than it was initially. The position of the control rods is the same as in the beginning.

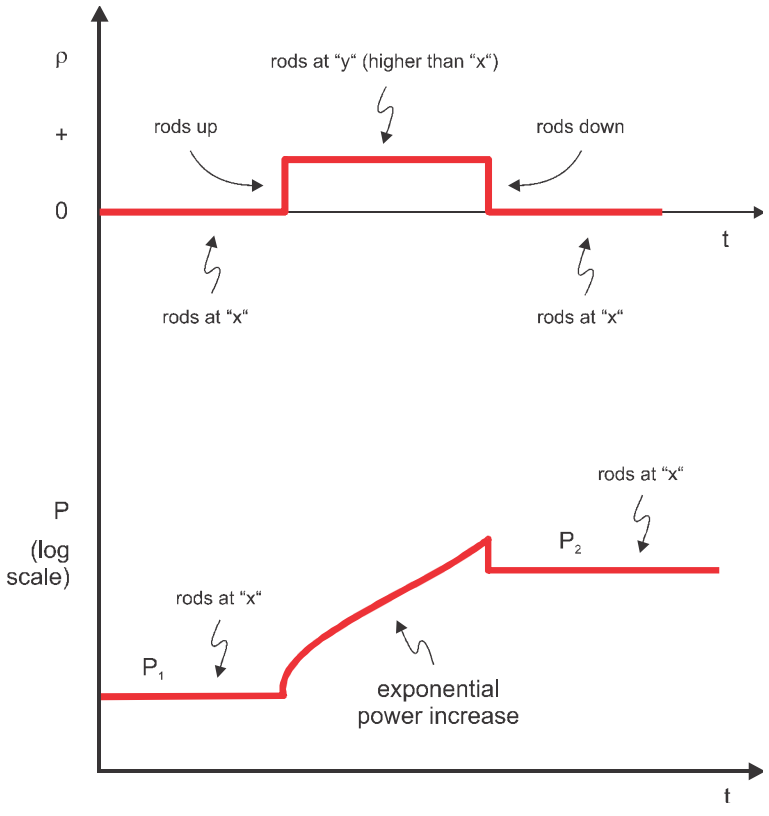


Figure 8.7: Reactor response to a positive step change in reactivity.

Figure 8.8 shows the reactor power response to a step insertion of negative reactivity which is performed by lowering the control rods a few steps into the core. The reactor was previously operating at constant power. After the transient, its power falls at a stable negative

period. After raising the control rods to their initial position, reactivity goes back to zero. Power stabilizes at a new value which is lower than it was initially.

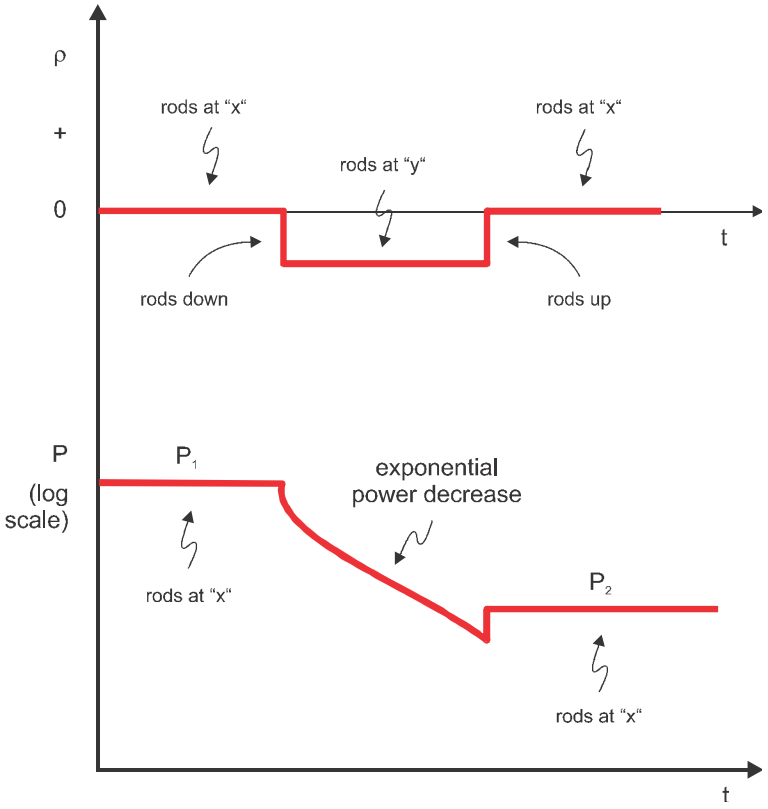


Figure 8.8: Reactor response to a negative step change in reactivity.

8.5 Questions

1. How is the reactor period, *T*, defined?
2. What does the reactor start-up rate or *SUR* tell us?
3. What is the reactor period, *T*, if *SUR* is equal to 0.25 dpm?
4. When is a reactor prompt critical?
5. Is the multiplication factor *k*, when reducing power, equal to 1, smaller than 1 or larger than 1?
6. Is *SUR* during constant power operation equal to 0, smaller than 0 or larger than 1?
7. What is the reactivity, ρ, when the reactor operates at stable 100% power?
8. In the first case, positive reactivity ρ = 20 pcm is inserted into the core, and in the second case, positive reactivity ρ = 50 pcm is inserted into the core. In which case does reactor power change at a shorter period?

9 REACTIVITY CHANGES

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Give a qualitative explanation of the temperature coefficients of reactivity.*
2. *Describe the impact of power change on reactivity.*
3. *Sketch reactor power response to a step change in reactivity.*
4. *Describe the formation and removal of ^{135}Xe and ^{149}Sm.*
5. *Sketch the reactivity due to Xe and Sm as a function of time after start-up.*
6. *Sketch the reactivity due to Xe and Sm after shutdown.*
7. *Sketch how the critical concentration of boron, C_B, changes relative to its burn-up rate.*
8. *Describe the principle of reactor control.*

The reactor core is subject to various influences which modify its properties. As its properties change, so does its reactivity. Based on how fast the core properties or its reactivity change during reactor operation, reactivity changes are classified as short-term, medium-term or long-term changes.

9.1 Short-term reactivity changes

When reactor power is modified in the operating power range from 0% to 100%, the fuel and coolant temperatures change. As the temperature changes, so do the properties of the core and its reactivity. Since changing reactor power is a relatively quick process, we speak of short-term reactivity changes.

Impact of coolant (moderator) temperature on reactivity

The moderator temperature coefficient, α_m, is defined as the change in reactivity per degree change in the average moderator temperature.

An increase in the average moderator temperature decreases the moderator density. This density decrease affects both the water's moderating and absorbing abilities and has the following consequences for the reactivity:

- The mass of water in the core decreases, while the mass of fuel remains constant. Consequently neutron absorption in the water decreases and absorption in the fuel increases. The thermal utilization factor, *f*, increases. This is a positive contribution to reactivity.
- As the water density decreases, the slowing down distance of neutrons increases, which is why the resonance absorption increases. The resonance escape probability, *p*, decreases, which means a negative contribution to core reactivity.

Whether α_m will be negative or positive depends on which of the two effects prevails. Pressurized light-water reactors are usually constructed for a negative moderator temperature coefficient.

Impact of fuel temperature on reactivity

The fuel temperature coefficient, α_f, is defined as the change in reactivity per degree change in the fuel temperature.

An increase in fuel temperature increases the resonance absorption of neutrons. Pressurized water reactors have just a few percent of fissile material, which is why parasitic resonance absorption predominates. The fuel temperature coefficient is negative.

Impact of void fraction on reactivity

The void coefficient, α_v, is the reactivity change per percent change in the void fraction.

An increase in the fraction of voids (steam bubbles) in the core reduces the moderator density. The impact on reactivity is similar to the impact of the moderator temperature. Due to the small total void fraction formed in the core, the associated total reactivity change is small.

Impact of power change on reactivity

The power coefficient, α_p, is defined as the change in reactivity per percent power change.

A change in power changes the coolant and fuel temperatures and the void fraction in the core. The combined effects of moderator temperature changes, fuel temperature changes and axial power redistribution are accounted for in the total power coefficient.

In pressurized water reactors the power coefficient is always negative. Increasing power inserts negative reactivity into the core and decreasing power inserts positive reactivity.

The **power defect** tells us how much core reactivity changes if reactor power is changed by a given value of ΔP.

In other words, if we increase reactor power by a given value of ΔP, the temperature effects will cause core reactivity to decrease by a value of $\Delta\rho$. If the power decreases, the temperature effects will cause core reactivity to increase.

Response to a step change in reactivity at operating power

At operating power levels, there are temperature feedback effects which were disregarded for lower power levels. Let us look at the response of a reactor operating at constant power if a given positive reactivity is added to it. Our experiment will not take into account the changed concentration of nuclides that affect reactor operation (for instance ^{135}Xe); we will assume that this transient is of short duration. We also assume that the power on the secondary side increases

correspondingly. As seen earlier, the easiest way for an operator to do this is to withdraw the control rods from the core a few steps (Fig. 9.1). Power starts increasing now that core reactivity is positive. The increasing power causes an increase in the fuel and moderator temperature, which is why negative reactivity is inserted into the core, reducing the initial positive reactivity inserted by withdrawal of the control rods. Power stabilizes at a new higher level when the negative reactivity (power defect) inserted by the power increase balances out the initial positive reactivity inserted by the control rods. At the end we have the reactor operating at higher power with the control rods withdrawn from the core a few steps relative to their position at the beginning of the transient. To increase reactor power in the power range where feedback temperature effects occur, positive reactivity should be added by control rods withdrawal or by other external means. Power is decreased by adding negative reactivity into the core.

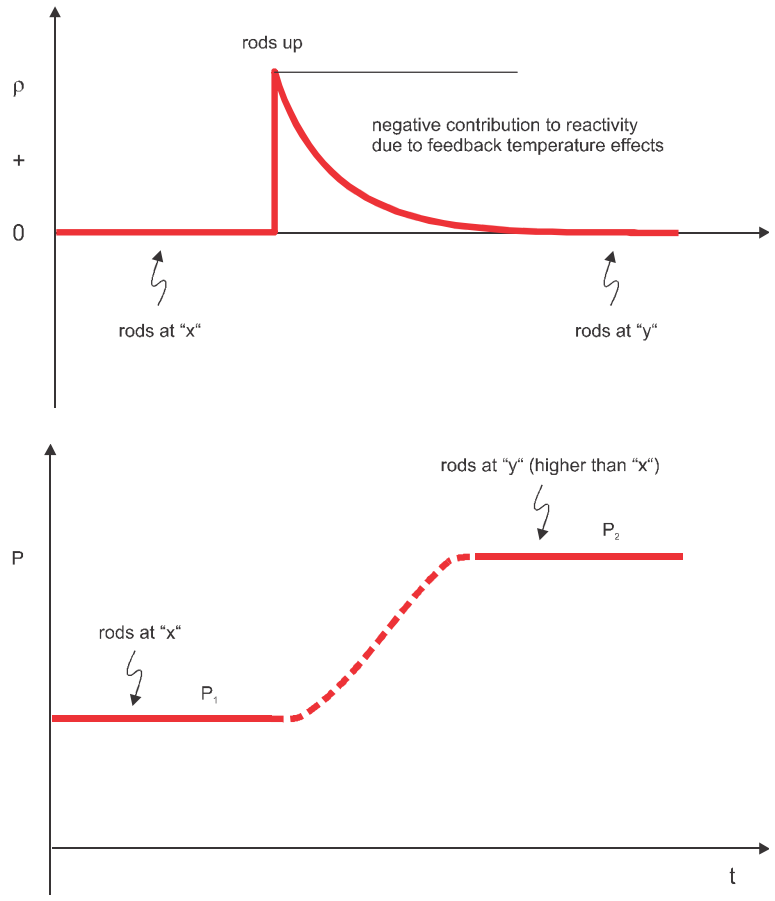


Figure 9.1: Reactor response to a reactivity increase during power operation.

Rod manipulation is not the only way to modify power in a pressurized water power plant. This can also be done by changing the concentration of boron in the reactor coolant.

9.2 Mid-term changes of reactivity

Fissions in the core result in over 200 different fission products. Most of them have small absorption cross sections or decay very quickly, which makes them irrelevant in terms of operation. However, two of these nuclides have very large absorption cross section for thermal neutrons and directly impact reactor operation. These are ^{135}Xe with a thermal neutron absorption cross section $\sigma_a = 2 \cdot 10^6$ b and ^{149}Sm with a thermal neutron absorption cross section $\sigma_a = 4 \cdot 10^4$ b.

Xenon, ^{135}Xe

^{135}Xe is produced:

- as a direct product of fission,
- from the decay of ^{135}I, which decays with a half-life of 6.6 hours and is the daughter product of the fission fragments ^{135}Sb and/or ^{135}Te:

$$^{135}\mathrm{Sb} \xrightarrow{\beta^-} {}^{135}\mathrm{Te} \xrightarrow{\beta^-} {}^{135}\mathrm{I} \xrightarrow{\beta^-} {}^{135}\mathrm{Xe}$$

^{135}Xe is removed:

- by burn-up via the neutron capture reaction:

$$^{135}\mathrm{Xe}(n, \gamma)^{136}\mathrm{Xe} ,$$

- ^{135}Xe decays with a half-life of 9.1 hours:

$$^{135}\mathrm{Xe} \xrightarrow{\beta^-} {}^{135}\mathrm{Cs} \xrightarrow{\beta^-} {}^{135}\mathrm{Ba} .$$

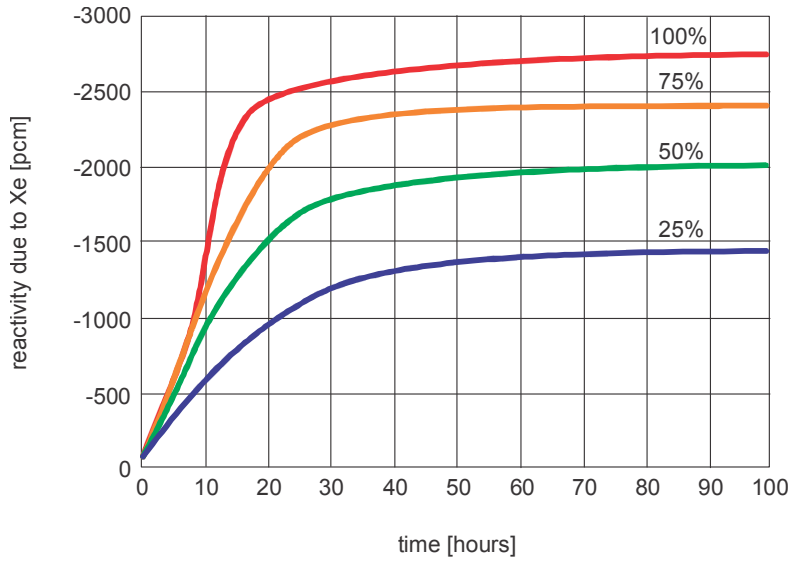


Figure 9.2: Reactivity due to ^{135}Xe in the reactor after start-up at different power levels.

After reactor start-up, the concentration of ^{135}Xe in the core starts to

grow, because the rate of its production from fission and from the radioactive decay of ^{135}I is higher than its rate of removal by burn-up and radioactive decay. When the production and removal rates balance out, xenon concentration reaches its equilibrium value (Fig. 9.2).

The equilibrium concentration of xenon is achieved approximately 40 to 50 hours after the initial power increase. The time depends on reactor power, but this dependence is not linear.

^{135}Xe production inserts negative reactivity into the core. In order to maintain reactor power after the initial power increase, negative reactivity due to xenon must be compensated by withdrawal of the control rods or by reducing the concentration of boric acid.

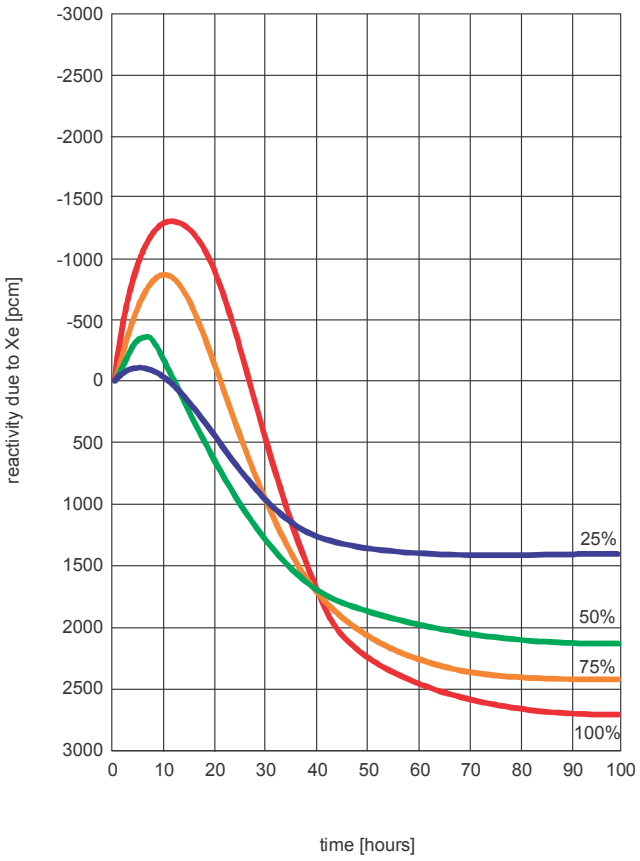


Figure 9.3: Change in the reactivity due to ^{135}Xe after shutting down a reactor which previously operated at power *P*.

After reactor shutdown, the neutron density in the core falls virtually to zero, which stops the formation of ^{135}Xe from fissions and its burn-up by neutron absorption. ^{135}Xe is produced with the decay of ^{135}I in the core and removed by means of its decay into ^{135}Cs. Since the half-life of ^{135}I is shorter that the half-life of ^{135}Xe, the concentration of xenon will rise immediately after shutdown and exceed its equilibrium concentration during power operation. The maximum ^{135}Xe concentration depends on the reactor power before shutdown and is reached in approximately 9 hours (also related to power). After this,

the xenon concentration starts to fall; after 24 hours, it is roughly equal to its level at shutdown, and after approximately 80 to 90 hours, ^{135}Xe is practically no longer present in the core (Fig. 9.3).

As ^{135}Xe decays, positive reactivity is inserted into the core. The operator must be aware of this and supply enough negative reactivity to prevent the recurrence of reactor criticality due to xenon decay.

Samarium, ^{149}Sm

^{149}Sm results from the decay of ^{149}Pm (promethium) with a half-life of 53.1 hours in the decay chain of the fission fragments ^{149} Ce and/or ^{149}Nd:

$$^{149}Ce \xrightarrow{\beta^-} {}^{149}Pr \xrightarrow{\beta^-} {}^{149}Nd \xrightarrow{\beta^-} {}^{149}Pm \xrightarrow{\beta^-} {}^{149}Sm.$$

^{149}Sm is removed by the neutron capture reaction:

$$^{149}\text{Sm}(n, \gamma)\,^{150}\text{Sm}\ .$$

When a reactor starts operating on power with a fresh core, the concentration of ^{149}Sm starts to increase. Its concentration reaches an equilibrium value after approximately 400 hours of operation and is independent of reactor power (Fig. 9.4).

As the concentration of ^{149}Sm grows, negative reactivity is inserted into the core and this reactivity must be compensated for. The change in reactivity due to ^{149}Sm is hardly perceptible because the process is very slow.

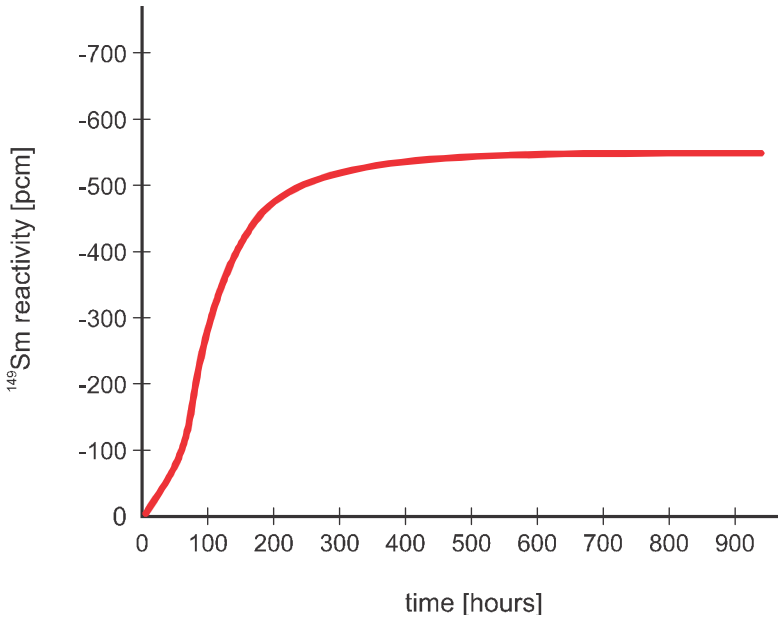


Figure 9.4: ^{149}Sm reactivity after startup of a reactor with a fresh core.

After reactor shutdown, the neutron density falls practically to zero.

^{149}Sm burnup by neutron absorption ceases. The core still contains ^{149}Pm, which decays into ^{149}Sm, so the concentration of ^{149}Sm starts to grow after shutdown and reaches its highest equilibrium value after approximately 400 hours; the concentration depends on the power prior to shut-down. This inserts additional negative reactivity into the core (Fig. 9.5). As the reactor is restarted on power, the equilibrium concentration of ^{149}Sm equal to its value prior to shut-down is re-established.

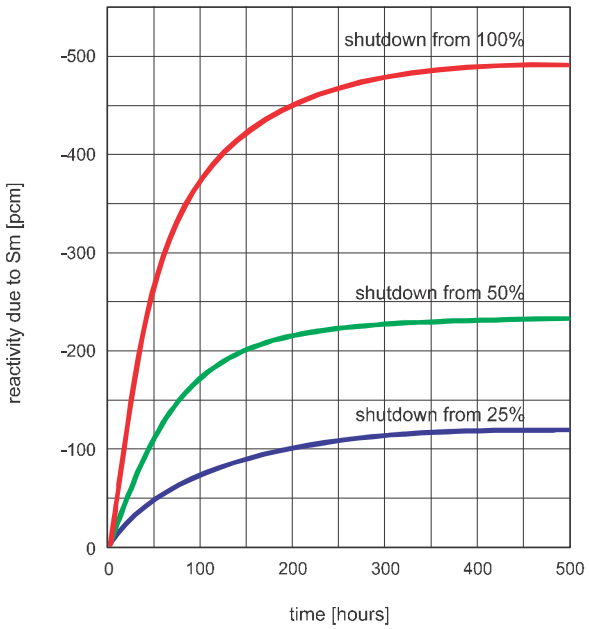


Figure 9.5: Change in ^{149}Sm reactivity after reactor shut-down.

9.3 Long-term changes of reactivity

With operation, the core of a light water reactor contains less and less fissile material. The core accumulates fission products, some of which have relatively large neutron absorption cross sections. Fuel burn-up is displayed as a decrease in core reactivity.

In pressurized water reactors, fuel is changed every 12 to 18 months. To sustain power plant operation until the next loading, it is necessary to ensure an initially higher core reactivity than needed for full power operation. This surplus of reactivity is called **excess reactivity**.

Excess reactivity is defined as the amount of surplus reactivity over that needed to enable reactor operation at zero power.

Excess reactivity is compensated by adding boron to the coolant and by burnable poisons. Since core reactivity decreases with burn-up, the concentration of boron in the moderator is gradually reduced over the fuel cycle. Fig. 9.6 shows a typical course of critical boron

concentration, C_B. The critical boron concentration (C_B) is defined as the concentration of boron required for the reactor to be critical on full power with the control rods withdrawn.

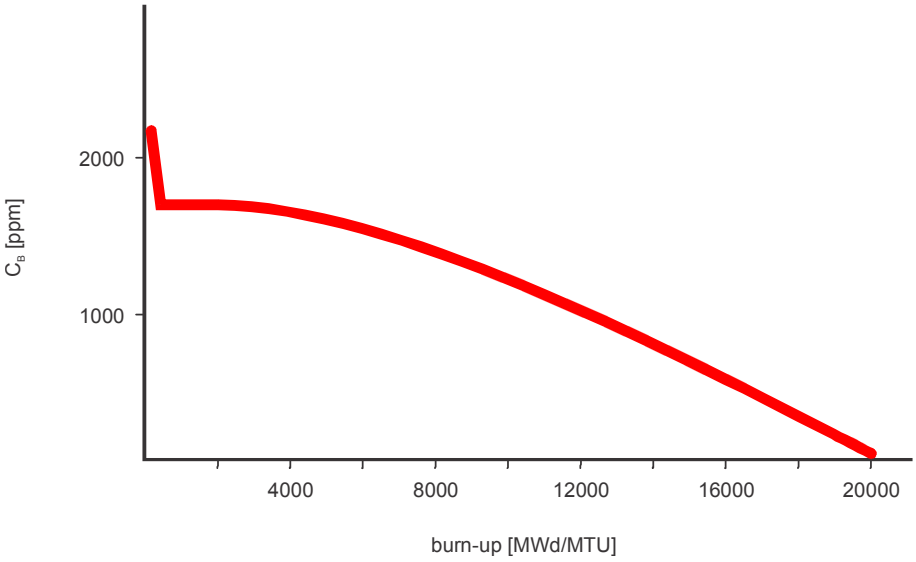


Figure 9.6: Typical course of critical boron concentration over the fuel cycle.

9.4 Reactor control

Operators primarily influence core reactivity in two ways:

a) **Control rods**: made from an alloy of 80% Ag, 15% In and 5% Cd, they are good absorbers of thermal and epithermal neutrons. The control rods are inserted into the core to increase non-fuel absorption of neutrons and as a result reduce the multiplication factor or core reactivity. The control rods are used to particularly compensate short-term changes in reactivity and to ensure a negative reactivity reserve needed for a reactor shut-down.

b) **Changing the boric acid concentration** in the primary coolant. Boron is a good absorber of thermal neutrons. Increasing the concentration of boron in the coolant leads to a relative increase in non-fuel absorption of neutrons. This lowers the multiplication factor, k, or core reactivity. Boron in water is used to particularly compensate long-term and medium-term changes in reactivity.

9.5 Questions

1. What does the moderator temperature coefficient, α_m, tell us?
2. What does the fuel temperature coefficient, α_g, tell us?
3. The power coefficient, α_p, in pressurized water reactors is always positive or negative?
4. If the power coefficient is α_p, = -15 pcm/%, how much does the core reactivity, ρ, change if the power is decreased by 2%?

5. What does the power defect tell us?
6. How is ^{135}Xe produced and how is it removed?
7. How does the concentration of ^{135}Xe in the core change following a reactor shut-down?
8. How is ^{149}Sm produced and how is it removed?
9. Does the equilibrium concentration of ^{149}Sm depend on the power at which the reactor is operating?
10. How is fuel burn-up and core poisoning compensated during the operation of pressurized water power plants?

10 SUBCRITICAL MULTIPLICATION

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Explain subcritical multiplication.*
2. *Define the subcritical multiplication factor, M.*
3. *Explain the 1/M curve for core loading.*

10.1 Neutron sources

The neutron population in the core is measured by the excore neutron detectors which are positioned outside the reactor vessel. They measure the neutrons that leak out of the core.

When a reactor is shut down, it is subcritical. The neutron population is low. We need to install an additional independent source of neutrons into the core to that already present (i.e. spontaneous fission of uranium), to get an adequate response on the excore neutron detectors. Independent neutron sources are needed to have:

- Control of the reactivity changes in the subcritical reactor,
- A signal on the detectors high enough to enable a controlled approach to criticality,
- Verification that the excore detectors are operational.

Neutron sources are regenerative or nonregenerative. Nonregenerative neutron sources are needed for initial reactor startup or after a long reactor shut-down. Examples of these include plutonium-beryllium (Pu-Be) source: (α particle emitted by Pu triggers the $^{9}Be(\alpha,n)^{12}C$ reaction) or ^{252}Cf (spontaneous fission). Regenerative sources which achieve full capacity after the reactor has been operating for some time (e.g. antimony-berillium (Sb-Be) source).

In reactors that have operated for several cycles, there is no need for additional neutron sources in the core. The fuel elements used in previous cycles provide a sufficient production of neutrons. This is because they contain the curium isotopes ^{242}Cm and ^{244}Cm which fission spontaneously and emit neutrons.

10.2 Approach to criticality

The number of neutrons per unit time in a subcritical reactor with an independent source is:

$$N = \frac{S_o}{1-k},$$

where S_o is the number of neutrons emitted by the source per time unit.

A subcritical reactor with an independent source creates an equilibrium neutron population, N, even when $k < 1$. The higher is k, the larger the equilibrium neutron population. The neutron population response to a step change in reactivity in a subcritical reactor during the approach to criticality is shown in Fig. 10.1 (lower part). If the independent source were removed from such a subcritical reactor, the neutron population would drop to zero.

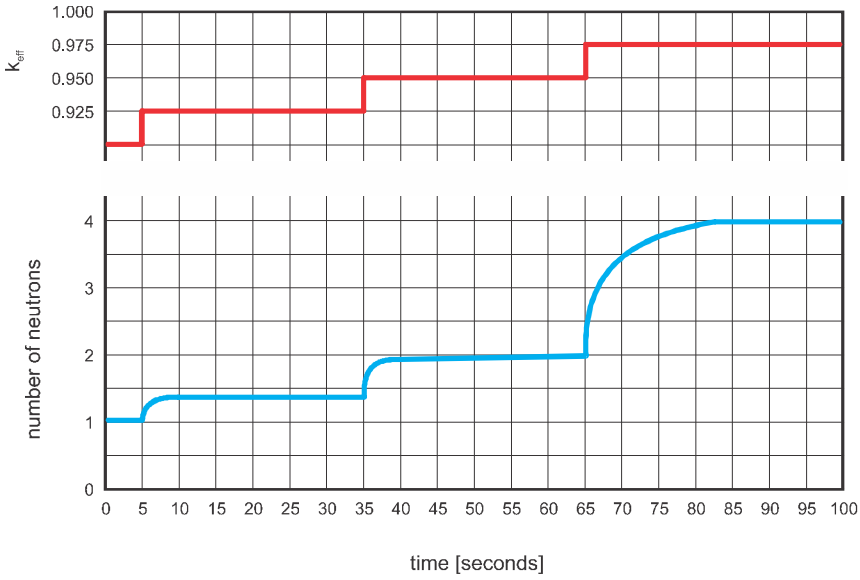


Figure 10.1: Neutron population upon a change in reactivity in a subcritical system.

Multiplication factor *M*

The multiplication factor M is defined as:

$$M = \frac{1}{1 - k} .$$

M is the ratio between the total neutron population per unit time and the number of neutrons per unit time emitted by the neutron source. M is a useful parameter for predicting criticality during core loading, namely:

$$k \to 1 \Rightarrow M \to \infty.$$

Since the value of M is undefined for $k = 1$, we prefer to use the factor $1/M$:

$$k \to 1 \Rightarrow \frac{1}{M} \to 0 .$$

When the reactor approaches criticality, $1/M$ approaches zero. $1/M$ graphs are used to predict the criticality of the reactor. Criticality can be achieved by inserting fuel elements into the core or by control rod withdrawal. In this case we plot $1/M$ as a function of the number of inserted fuel elements or of the control rods position. Criticality is predicted where the $1/M$ graph intersects the abscissa ($1/M = 0$). Core loading should be managed with great care, and the neutron population must be continuously monitored.

Let us look at an example of how fuel is loaded into an empty core.

- In an empty core, $k = 0$. Detectors are employed to count c_o, the number of impulses per unit time (base count rate), which result exclusively from the independent sources. $1/M$ is calculated:

$$\frac{1}{M} = \frac{c_0}{c_0} = 1 .$$

  The value of $1/M$ is entered in the graph. $1/M$ is placed on the ordinate (vertical axis) and the number of inserted fuel elements on the abscissa (horizontal axis).
- We add a few fuel elements to the core. This makes $k \neq 0$. The detector counts c_1 impulses:

$$\frac{1}{M} = \frac{c_0}{c_1} < 1 .$$

  We draw a line to connect the resulting point and extend it to the horizontal axis. This gives an estimate of how many more fuel elements will be needed to reach critical loading or how close to criticality we are. We insert further fuel elements into the core and calculate $1/M$. The process is repeated until the core is filled with fuel. This imaginary example is covered in Table 10.1 and Fig. 10.2.

Table 10.1: Values obtained in calculating $1/M$.

step	Number of inserted elements	c_i	c_0/c_i
0	0	100	1
1	10	125	0.8
2	20	166	0.6
3	30	250	0.4
4	40	500	0.2
5	45	1000	0.1

For such a prediction it is not necessary to start the measurement with an empty core. The base count rate can be placed anywhere. In this case the $1/M$ curve is renormalized to a new base count. In this case we rather speak of the ICRR (Inverse Count Rate Ratio) curve instead

of 1/*M* curve. Prediction of the criticality (ICRR = 0) remains the same.

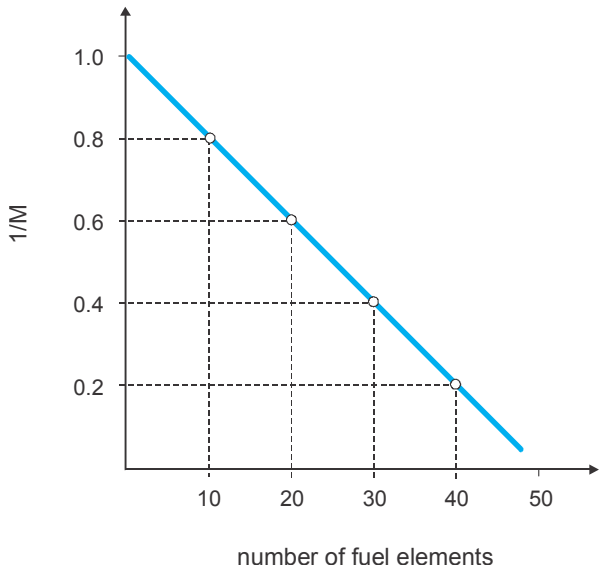


Figure 10.2: Core loading procedure.

When loading the core in power reactors, ICRR is identified after inserting each fuel element. If the factor ICRR deviates abnormally, we must stop all activities in the core and act in accordance with the relevant procedures for such cases. For safety reasons, the core of power reactors is poisoned with neutron absorbers (boric acid) before loading to such a degree that it is still heavily subcritical once loading is completed (Fig. 10.3).

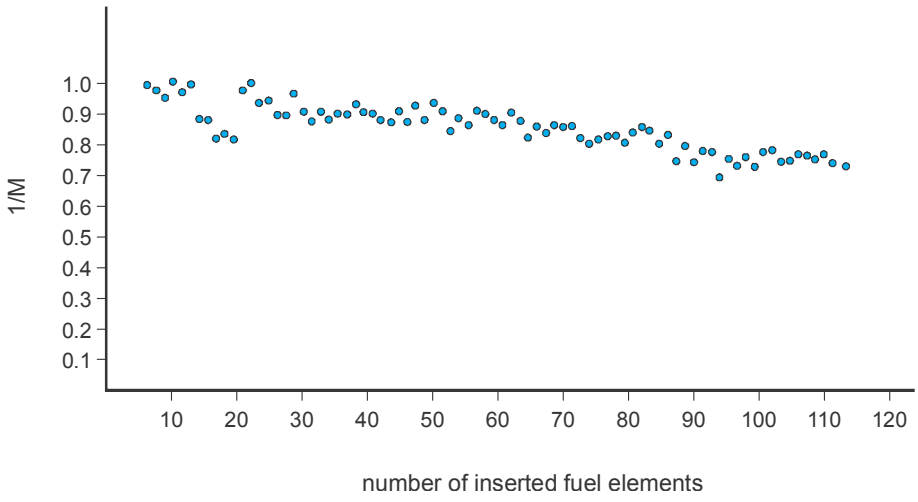


Figure 10.3: A typical PWR loading process.

10.3 Questions

1. Why are neutron sources inserted into the core?

2. During the approach to criticality (k=1) in the source range, to what value does the 1/M curve approach?
3. During the approach to criticality in the source range (subcritical multiplication), in which case are the changes in SR detector response greater: when the multiplication factor, k, changes from 0.9 to 0.925, or when the multiplication factor, k, changes from 0.925 to 0.95?

11 HEAT REMOVAL FROM NUCLEAR REACTORS

Learning objectives

After completing this chapter, the trainee will be able to:

1. *Name three modes of heat transfer.*
2. *Describe heat conduction.*
3. *Describe forced and natural convection.*
4. *Describe heat transfer to a fluid with phase change.*
5. *Explain the formation of steam bubbles.*
6. *Sketch and describe the boiling curve.*
7. *Define the boiling crisis.*
8. *Describe the average and maximum thermal power of a fuel rod in a PWR.*
9. *Describe hot channel factors and explain the reasons for their limits.*

To understand the heat processes ocurring in a nuclear reactor it is important to know the rate and modes of heat transfer between the source of heat and the system, or the system and the environment. *Heat* is **energy which passes from a point of higher temperature to a point of lower temperature**. If there is no difference in temperature, there can be no heat transfer.

The measure of heat flow rate is **heat flux**. It is defined as the quantity of heat per unit time, which means it is measured in **J/s** or **W**.

$$\dot{Q} = \frac{Q}{\Delta t}.$$

In a technical context, we often refer to heat flux density, which means heat flux per unit surface area and is expressed in **W/m^2**.

Heat can be transferred from points of higher temperature to points of lower temperature in three distinct ways:

- heat conduction through matter (diffusion),
- heat transfer by means of fluid flows (convection),
- radiation.

11.1 Heat conduction

Heat conduction is a process which takes place at the atomic level. If we hold a cold metal rod in our hand and place the other end in a flame, we will soon feel a rise in temperature in our hand. In microscopic terms, energy passes from the hot tip of the rod to its cold tip by means of the vibrations of atoms and electrons in the metal. Heat conduction through a flat wall of homogeneous material (Fig.

11.1) is macroscopically described by the simple equation:

$$\dot{Q} = \frac{\lambda \, A \, \Delta \, T}{\Delta \, x} \, ,$$

where the symbols mean:

λ	heat conductivity	[W/mK],
$\boldsymbol{A}$	surface area of the wall	[m^2],
$\Delta \boldsymbol{T}$	temperature difference	[K],
$\Delta \boldsymbol{x}$	wall thickness	[m].

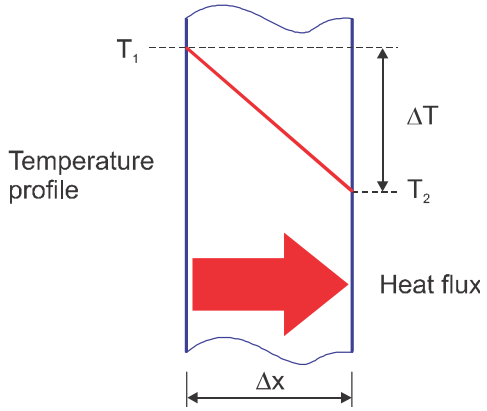


Figure 11.1: Heat conduction and temperature profile in a flat wall.

Thus, the better the wall material conducts heat, the larger the wall area, the larger the temperature difference and the thinner the wall, the greater is the resulting heat flux through the wall. Heat conductivity, $\boldsymbol{\lambda}$, is a property of a substance. Good heat conductors, e.g. metals, have a large $\boldsymbol{\lambda}$, whereas heat insulators such as gases, wood, brick etc. have a small $\boldsymbol{\lambda}$.

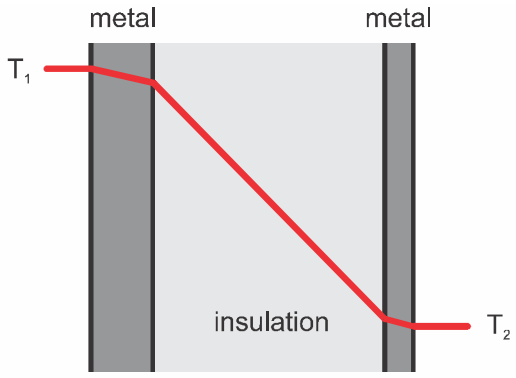


Figure 11.2: Temperature profile in a wall made of various materials (e.g. metal – insulation – metal).

The temperature profile shows the temperature gradient during heat conduction through the wall. In good conductors (e.g. metals) the

profile has a gentle slope and in poor conductors (e.g. ceramics, insulating materials) it is steep. If a wall consists of different materials (e.g. metal piping with a layer of thermal insulation and a metal outer jacket, which is a common case in thermal installation), its temperature profile is roughly as shown Fig. 11.2.

11.2 Convection

Convection involves heat transferred by the flow of fluid. Convection can be natural or forced.

When a fluid moves due to a difference in fluid density, we speak of **natural convection**. The difference in density can be caused by temperature differences or changes, and also by phase changes. For example; a warm layer of fluid gains buoyancy and rises, is replaced by the cooler surrounding fluid which then heats up again, spreads out and cools. The process continues as long as there is a heat source and a heat sink.

An example of **natural convection due to a fluid density difference** is the cooling of the reactor core after turning off the pressurized water reactor pumps. The source of heat (reactor) is the lowest point in the primary system. The water inside heats up on contact with the fuel elements, which reduce its density. The water, which now has buoyancy, flows up into the steam generators, which are located above the reactor and act as heat sinks. The primary water cools in the steam generators, its density increases and the water sinks back into the reactor beneath the generators. In this case, the primary system works like a central heating system with natural circulation, where the reactor has the role of the stove and the generators have the role of radiators. We should always keep in mind that the **existence** of natural circulation **requires a heat source and heat sink**.

An example of **natural convection due to a phase change** is the circulation of secondary water inside a steam generator (S/G). Secondary water is at the boiling point temperature throughout the steam generator. In the riser (the upward flow channel between the U-tubes) the secondary water contains many steam bubbles due to boiling. The density of the resulting two-phase flow in the riser is lower than the water density in the downcomer (the downward flow channel – annular space between the S/G pressure shell and the U-tube mantle) which causes it to rise.. At the top of the S/G, the bubbles separate from the water, and the boiling water with a density greater than the two-phase flow density returns to the bottom of the steam generator through a downcomer.

Forced convection means that a fluid is forced to move along a heating surface by means of a pump or fan. Usually, the fluid

velocities are significantly higher than in natural convection. An example of forced convection at a PWR is the flow of the primary coolant through the core driven by the operation of reactor coolant pumps.

The heat flux transferred from wall to fluid can be calculated by using the following equation:

$$\dot{Q} = h_c \, A \, \Delta T ,$$

where:

- h_c is the heat transfer coefficient,
- A is the wall surface area,
- ΔT is the temperature difference between the wall and the fluid.

We can see that, like in heat conduction, the heat flux is proportional to the wall area, temperature difference and heat transfer coefficient. It is not as easily determined as in the case of heat conduction, because it depends on a greater number of parameters: fluid velocity, pressure, temperature, the flow regimes of potential two-phase flow, etc.

11.3 Radiative heat transfer

Radiation is the emission of internal energy by electromagnetic waves. Radiation can spread through empty space (a vacuum), as well. The heat flux emitted by a hot body is proportional to its absolute temperature to the fourth power (Stefan-Boltzmann law):

$$\dot{Q} = \varepsilon \, \sigma \, A \, T^4 ,$$

where:

- ε is the emission coefficient (1 for a black body),
- σ is the Stefan-Boltzmann constant,
- A is the total area of the body radiating heat,
- T is the absolute temperature.

An everyday example of radiative heat transfer is solar radiation. An example of radiative heat transfer in the reactor core would be rather unpleasant. If the reactor vessel was left completely without water in the event of a large break loss of coolant accident, radiative heat transfer would be the only option for removing decay heat. This would entail a high core temperature, which means overheating the fuel rods and melting of the core.

11.4 Boiling heat transfer

When heat is transferred from a solid body to a liquid, the liquid may boil. This is **heat transfer with phase change**. Even for a liquid which is only partially boiling, the heat flux is much larger than in natural or forced convection, which also means better heat transfer.

Saturated boiling occurs when water is at the boiling point temperature throughout its bulk. If the bulk water temperature is below the boiling point, **subcooled boiling** may occur. Bubbles form in small hollows in the wall of the solid body (higher local temperature) and break away when buoyancy overcomes the force which holds them to the wall. A growing bubble pushes aside water and when it breaks away, the surrounding water rushes in to take its place (Fig. 11.3). In subcooled water the bubbles collapse, but in saturated boiling they form an element of the two-phase flow.

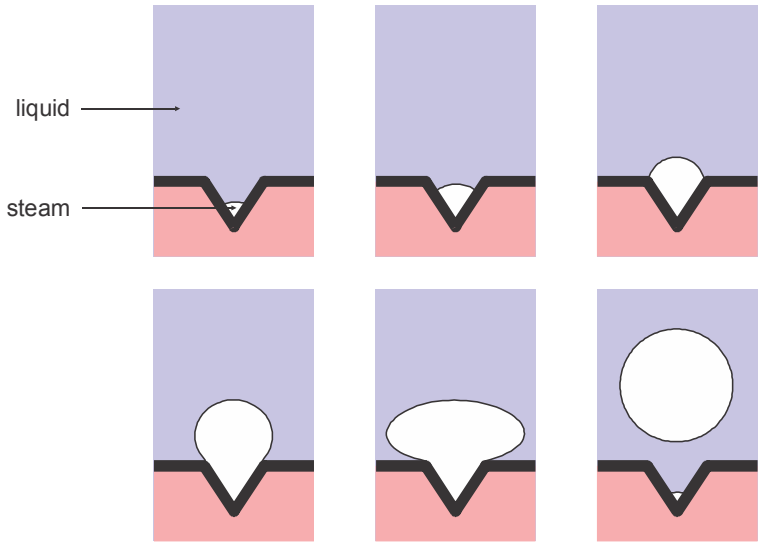


Figure 11.3: Bubble formation.

Simple observation in daily life reveals that there are very different types of boiling: the almost imperceptible boiling of gently simmering soup, vigorous bubbling when we forget to reduce the flame beneath the pot on a gas cooker, a drop of water which sizzles on a red-hot burner or hot plate, etc. In a technical context, different types of boiling are highly important and should be defined in more detail. Let us examine how the heat flux and temperature difference between water and the heating surface are related.

The boiling curve (Fig. 11.4) shows the dependence of heat flux density on the temperature difference between the wall (heat transfer surface) of a solid body (T_W) and the boiling point of water (saturation temperature - T_S). To make the curve easier to represent on a graph, the scales on both axes are logarithmic.

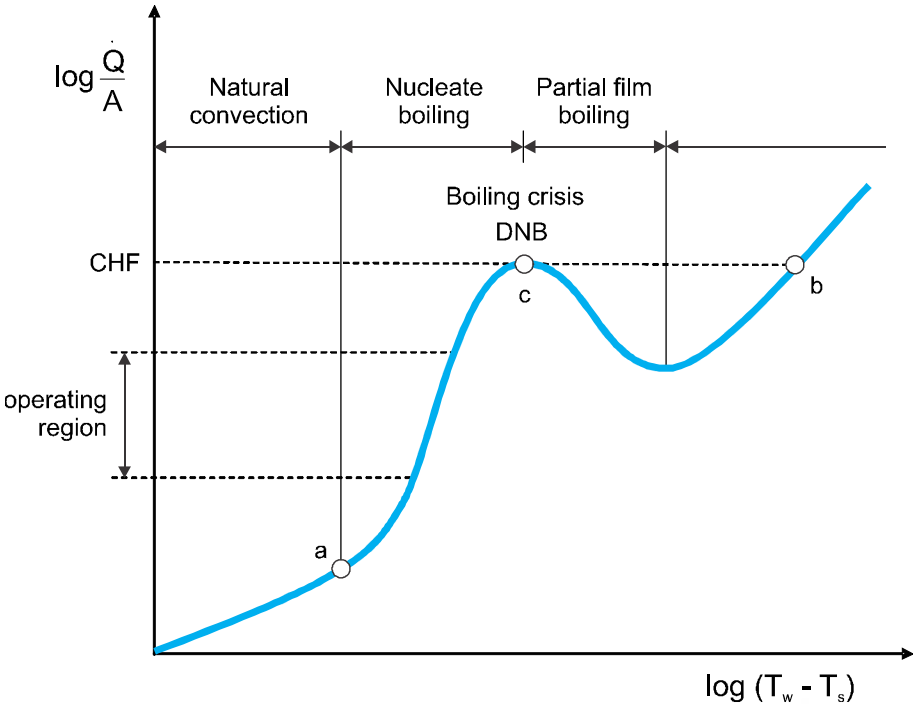


Figure 11.4: Boiling curve.

A typical boiling curve can be divided into 4 regions:

- **Natural convection** – Occurs at a small temperature difference (T_W - T_S).
- **Nucleate boiling** – Point **a** represents the beginning of boiling. When the wall temperature exceeds the saturation temperature steam bubbles start to form on the surface. Bubbles detach themselves from the wall, induce fluid mixing and substantially increase the heat flux density. If the bulk fluid temperature is below the saturation temperature the process is called subcooled nucleate boiling. When the bulk fluid temperature is at the saturation temperature, it is called saturated nucleate boiling.
- **Partial film boiling** – Bubbles combine into layers next to the wall of the heating body. The layer of vapour insulates the surface and reduces heat transfer. In this region, the vapour layers are not stable. They are washed away by fluid, but new thin layers (films) of vapour instantly form next to the heating body.
- **Film boiling and radiation** – Stable layers of vapour form next to the heating body, resulting in poor heat transfer despite a relatively large temperature difference. At very high heating body temperatures, heat is also transferred by electromagnetic radiation.

Point ***c*** on the curve is called the boiling crisis or Departure from Nucleate Boiling – DNB. At this point, nucleate boiling switches over to film boiling. The heat flux at point ***c*** is called the critical or DNB heat flux. The state at point ***c*** is not stable. The same heat flux could exist at point ***b*** which indicates radiative heat transfer. When the boiling crisis is reached, the temperature of the heating body

surrounded by an insulating vapour film can rise rapidly (leap from point ***c*** → ***b***), which signifies burnout of the heating body.

The heating bodies in water-cooled nuclear reactors are the fuel rods. At each point of the reactor core the heat flux must always be smaller than the critical heat flux to avoid dangerous damage to fuel rod cladding or fuel melting.

The ratio between the critical and actual heat flux is called the critical heat flux ratio or DNBR – Departure from Nucleate Boiling Ratio:

$$DNBR = \frac{\text{critical heat flux}}{\text{actual heat flux}} .$$

The DNBR should always be greater than 1. Its limit value is set down in the Technical Specifications for each individual nuclear power plant.

11.5 Relation to reactor core safety limits

During normal operation and during incidents, the integrity of the reactor core must be maintained. This means that the fuel pellets must not melt and that the cladding must not suffer damage. Core integrity in the case of normal operation and moderately frequent incidents is ensured by the design limits which relate to fuel pellet temperature and the boiling crisis. The limits would be relatively easy to define if the power distribution in the core were even. Unfortunately it is not. The power distribution is uneven and depends on the nuclear, thermodynamic and fabrication parameters. A practical approach is to describe the power distribution by multiplying the average values by peaking factors.

Average linear power density is the thermal power released per unit fuel rod length (kW/m). It is calculated by dividing the nominal reactor thermal power by the total length of all the fuel rods.

Example:
The thermal power of a PWR is 1994 MW. There are 121 fuel elements in the core. Each fuel element consists of 235 rods with a length of 3.6 m. The fraction of heat released in the fuel is 97.4%. Calculate the average linear power density:
Answer:

$$\bar{q} = \frac{1994 \times 1000\,\text{kW} \times 0.974}{121 \times 235 \times 3.6\,\text{m}} = 18.7\,\text{kW/m} .$$

The linear power density is not equal for all the rods in a fuel element. It also varies among individual elements and even more with the height of a fuel rod in the core.

Factors that take account of the unevenness of the thermal power released are called peaking factors or **hot channel factors**. A peaking factor is the ratio between the highest value of an individual parameter and its average value.

The heat flux hot channel factor or heat flux peaking factor is defined as the ratio:

$$F_Q = \frac{\text{maximum heat flux in the core}}{\text{average heat flux in the core}}.$$

The maximum allowed heat flux hot channel factor is set in the operational limits (see Module 11). An example of the maximum F_Q, with respect to reactor power is shown on Fig. 11.5.

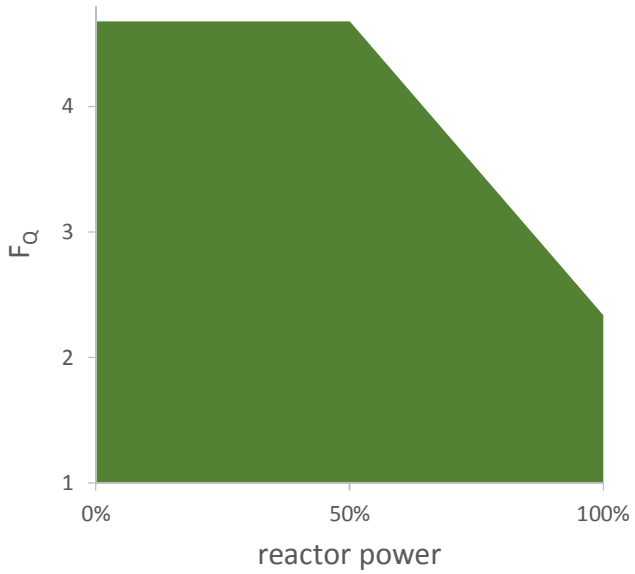


Figure 11.5: An example of the operational limit of the maximum heat flux hot channel factor.

Example:
For the reactor in the previous example, and given the operational limit from Fig. 11.5, the maximum allowed linear power density is:

$$q_{\max} = F_Q \cdot \bar{q} = 2.35 \cdot 18.7 \frac{kW}{m} = 44 \frac{kW}{m}.$$

Figure 11.6 shows the distribution of average and maximum temperatures in the cross-section of a fuel rod. The highest fuel temperature in the core develops in the middle of the fuel pellet and it is directly proportional to linear power (given an approximately constant coolant temperature).

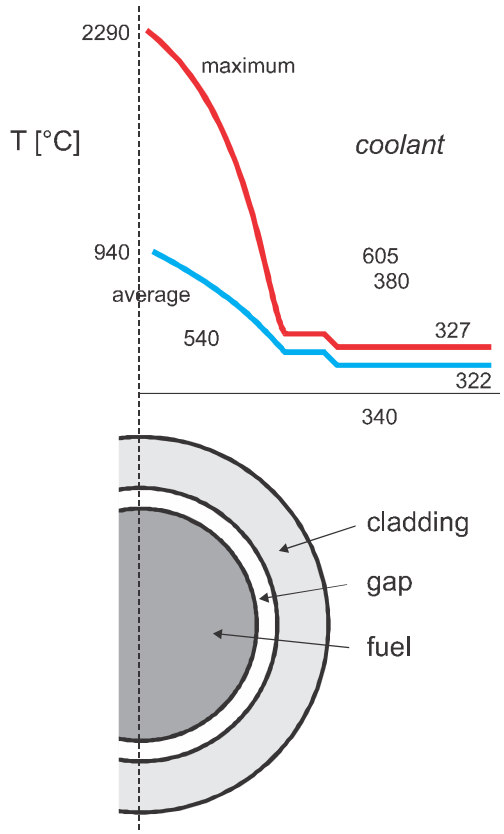


Figure 11.6: Temperature profile in a fuel rod.

The F_Q limit ensures that:
- the maximum temperature in the middle of a fuel pellet (centreline temperature) during normal operation is **lower** than the melting point temperature,
- the maximum temperature of fuel rod cladding during a loss of coolant accident is **lower** than 1200° C.

The enthalpy rise hot channel factor, $F_{\Delta H}$, gives the ratio between the total power of the hottest element and the average element:

$$F_{\Delta H} = \frac{\text{maximum fuel rod integral power}}{\text{average fuel rod integral power}} .$$

Hot channel factors are calculated on the basis of measurements of thermal neutron flux made with movable neutron detectors. These measurements are made periodically, as a rule every 1000 MWd/t of core burn-up, which is approximately once a month at a typical PWR.

In the meantime, the operational limits for hot channel factors are not crossed as long as the reactor operates so that:
- the axial flux difference (the difference between the power of the upper and bottom half of the core) is within the target band;
- different groups of control rods are withdrawn with a

programmed overlap,
- control rods are above the insertion limit,
- individual control rods in a group are withdrawn to the same height; on no account should they be more than a certain number of steps apart.

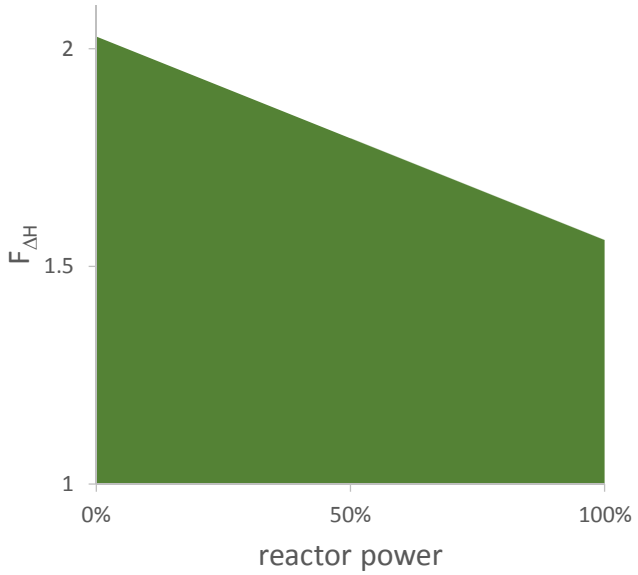


Figure 11.7: An example of the operational limit of the maximum enthalpy rise hot channel factor.

The $F_{\Delta H}$ limit ensures that a boiling crisis does not occur during normal operation and moderately frequent incidents.

The critical heat flux ratio or **Departure from Nuclear Boiling Ratio** (DNBR) is the ratio between the critical heat flux and the actual heat flux at any point in the core. During operation, DNBR is usually around 2.0.

In a typical PWR, DNBR should always be greater than 1.36. This value ensures with a 95% probability that a boiling crisis will not occur.

The critical heat flux ratio depends on primary cooling system parameters during operation.

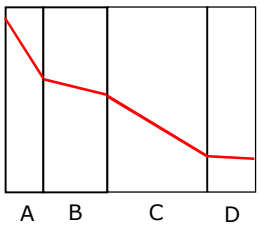
DNBR is reduced when:
a. the pressure in the primary system falls,
b. the flow in the primary system decreases,
c. the temperature of the primary coolant rises.

Since such operational effects can decrease the DNBR below the limit set down in the Technical Specifications, the reactor will perform an automatic shutdown when the above parameters reach specific values.

11.6 Questions

1. What is heat flux?
2. What does heat flux density mean?
3. Which of the materials in the flat wall is the best heat conductor?

A
B
C
D



Temperature profile in a flat wall made of four materials

4. What kind of heat transfer is convection?
5. What kind of heat transfer is natural convection? Is it possible in a PWR?
6. What kind of heat transfer is forced convection?
7. What is nucleate boiling? What does it mean for heat transfer?
8. What is the boiling crisis?
9. What is the consequence of the boiling crisis?

12 REFERENCES

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13 *Appendix A:* Periodic table of the elements

1 **H** *hydrogen* 1.0079																	2 **He** *helium* 4.0026
3 **Li** *lithium* 6.941	4 **Be** *beryllium* 9.0122											5 **B** *boron* 10.811	6 **C** *carbon* 12.011	7 **N** *nitrogen* 14.007	8 **O** *oxygen* 15.999	9 **F** *fluorine* 18.998	10 **Ne** *neon* 20.18
11 **Na** *sodium* 22.99	12 **Mg** *magnesium* 24.305											13 **Al** *aluminium* 26.982	14 **Si** *silicon* 28.086	15 **P** *phosphorus* 30.974	16 **S** *sulphur* 32.066	17 **Cl** *chlorine* 35.453	18 **Ar** *argon* 39.948
19 **K** *potassium* 39.098	20 **Ca** *calcium* 40.078	21 **Sc** *scandium* 44.956	22 **Ti** *titanium* 47.867	23 **V** *vanadium* 50.942	24 **Cr** *chromium* 51.996	25 **Mn** *manganese* 54.938	26 **Fe** *iron* 55.846	27 **Co** *cobalt* 58.933	28 **Ni** *nickel* 58.693	29 **Cu** *copper* 63.546	30 **Zn** *zinc* 65.409	31 **Ga** *gallium* 69.723	32 **Ge** *germanium* 72.64	33 **As** *arsenic* 74.922	34 **Se** *selenium* 78.96	35 **Br** *bromine* 79.904	36 **Kr** *krypton* 83.798
37 **Rb** *rubidium* 85.478	38 **Sr** *strontium* 87.62	39 **Y** *yttrium* 88.906	40 **Zr** *zirconium* 91.224	41 **Nb** *niobium* 92.906	42 **Mo** *molybdenum* 95.94	43 **Tc** *technetium* (98)	44 **Ru** *ruthenium* 101.07	45 **Rh** *rhodium* 102.91	46 **Pd** *palladium* 106.42	47 **Ag** *silver* 107.87	48 **Cd** *cadmium* 112.41	49 **In** *indium* 114.82	50 **Sn** *tin* 118.71	51 **Sb** *antimony* 121.76	52 **Te** *tellurium* 127.6	53 **I** *iodine* 126.9	54 **Xe** *xenon* 131.29
55 **Cs** *caesium* 132.91	56 **Ba** *barium* 137.33	57 **La*** *lanthanum* 138.9	72 **Hf** *hafnium* 178.49	73 **Ta** *tantalum* 180.95	74 **W** *tungsten* 183.84	75 **Re** *rhenium* 186.21	76 **Os** *osmium* 190.23	77 **Ir** *iridium* 192.22	78 **Pt** *platinum* 195.08	79 **Au** *gold* 196.97	80 **Hg** *mercury* 200.59	81 **Tl** *thallium* 204.38	82 **Pb** *lead* 207.2	83 **Bi** *bismuth* 208.98	84 **Po** *polonium* (209)	85 **At** *astatine* (210)	86 **Rn** *radon* (222)
87 **Fr** *francium* (223)	88 **Ra** *radium* (226)	89 **Ac**◊ *actinium* (227)	104 **Rf** *rutherfordium* (267)	105 **Db** *dubnium* (268)	106 **Sg** *seaborgium* (271)	107 **Bh** *bohrium* (272)	108 **Hs** *hassium* (270)	109 **Mt** *meitnerium* (276)	110 **Ds** *darmstadtium* (281)	111 **Rg** *roentgenium* (280)	112 **Cn** *copernicium* (285)						

Lanthanoids*	58 **Ce** *cerium* 140.11	59 **Pr** *praseodymium* 140.91	60 **Nd** *neodymium* 144.24	61 **Pm** *promethium* (145)	62 **Sm** *samarium* 150.36	63 **Eu** *europium* 151.96	64 **Gd** *gadolinium* 157.25	65 **Tb** *terbium* 158.93	66 **Dy** *dysprosium* 162.5	67 **Ho** *holmium* 164.93	68 **Er** *erbium* 167.26	69 **Tm** *thulium* 168.93	70 **Yb** *ytterbium* 173.04	71 **Lu** *lutetium* 174.97
Actinoids◊	90 **Th** *thorium* 232.04	91 **Pa** *protactinium* 231.04	92 **U** *uranium* 238.03	93 **Np** *neptunium* (237)	94 **Pu** *plutonium* (244)	95 **Am** *americium* (243)	96 **Cm** *curium* (247)	97 **Bk** *berkelium* (247)	98 **Cf** *californium* (251)	99 **Es** *einsteinium* (252)	100 **Fm** *fermium* (257)	101 **Md** *mendelevium* (258)	102 **No** *nobelium* (259)	103 **Lr** *lawrencium* (262)

26
Fe
iron
55.846

atomic number, Z
chemical symbol of the element
English name of the element
the element's relative atomic mass, A_r

The symbols for naturally occurring elements are in solid letters (**Na**), whereas hollow letters (Pu) denote artificially produced elements.

For radioactive elements, the mass number of the longest-lived isotope is given instead of the relative atomic mass.

14 *Appendix B:* Some important nuclide data

A selection of important nuclides in nuclear technology. Naturally occurring nuclides are in **bold**. The sign (+γ) means that, after α or β decay, the newly formed nucleus de-excites by emitting gamma radiation.

Z	nuclide	decay, branching ratio	isotopic abundance / half-life
0	^{1}n	β^-	10.3 min
1	**^{1}H**		**99.99%**
	^{2}H		**0.015%**
	^{3}H	β^-	12.3 y
2	**^{3}He**		**0.00014%**
	^{4}He		**~100%**
3	**^{6}Li**		**7.59%**
	^{7}Li		**92.41%**
4	**^{9}Be**		**100%**
5	**^{10}B**		**19.80%**
	^{11}B		**80.20%**
6	**^{12}C**		**98.89%**
	^{13}C		**1.11%**
	^{14}C	β^-	5730 y
7	**^{14}N**		**99.63%**
	^{15}N		**0.37%**
	^{16}N	β^- (+γ)	7.13 s
8	**^{16}O**		**99.76%**
	^{17}O		**0.04%**
	^{18}O		**0.20%**
11	^{22}Na	β^+ (+γ)	2.602 y
	^{23}Na		**100%**
	^{24}Na	β^- (+γ)	14.96 h
13	**^{27}Al**		**100%**
19	**^{39}K**		**93.26%**
	^{40}K	**β^- 89%** (+γ) **β^+ 11%**	**0.012%** **$1.27\cdot10^{9}$ y**
	^{41}K		**6.73%**
24	**^{50}Cr**		**4.35%**
	^{51}Cr	β^+ (+γ)	27.7 d
	^{52}Cr		**83.79%**
	^{53}Cr		**9.50%**
	^{54}Cr		**2.37%**
25	**^{55}Mn**		**100%**
	^{56}Mn	β^- (+γ)	2.58 h
26	**^{54}Fe**		**5.85%**
	^{55}Fe	β^+	2.737 y
	^{56}Fe		**91.75%**
	^{57}Fe		**2.12%**
	^{58}Fe		**0.28%**
	^{59}Fe	β^- (+γ)	44.5 d
27	**^{59}Co**		**100%**
	^{60}Co	β^- (+γ)	5.27 y
29	**^{63}Cu**		**69.17%**
	^{64}Cu	β^+ 61% (+γ) β^- 39%	12.7 h
	^{65}Cu		**30.83%**
35	**^{79}Br**		**50.69%**
	^{81}Br		**49.31%**
	^{87}Br	β^- 97,4% (+γ) β^-n 2,6%	56 s
36	**^{84}Kr**		**57.00%**
	^{85}Kr	β^-	10.8 y
	^{85m}Kr	β^- 79% γ 21%	4.48 h
	^{86}Kr		**17.30%**
	^{87}Kr	β^- (+γ)	1.27 h
	^{88}Kr	β^- (+γ)	2.84 h
38	**^{84}Sr**		**0.56%**
	^{86}Sr		**9.86%**
	^{87}Sr		**7.00%**
	^{88}Sr		**82.58%**
	^{90}Sr	β^-	28.8 y
53	**^{127}I**		**100%**
	^{129}I	β^- (+γ)	$1.57\cdot10^{7}$ y
	^{131}I	β^- (+γ)	8.02 d
	^{135}I	β^- (+γ)	6.57 h
54	**^{124}Xe**		**0.10%**
	^{126}Xe		**0.09%**
	^{128}Xe		**1.91%**
	^{129}Xe		**26.40%**
	^{130}Xe		**4.07%**
	^{131}Xe		**21.23%**
	^{131m}Xe	γ	11.9 d
	^{132}Xe		**26.91%**
	^{133}Xe	β^- (+γ)	5.243 d
	^{133m}Xe	γ	2.19 d
	^{134}Xe		**10.44%**
	^{135}Xe	β^- (+γ)	9.14 h
	^{136}Xe		**8.86%**
	^{137}Xe	β^- (+γ)	3.8 min
	^{138}Xe	β^- (+γ)	14.1 min
55	**^{133}Cs**		**100%**
	^{134}Cs	β^- (+γ) β^+ 0.0003%	2.066 y
	^{135}Cs	β^-	$2.3\cdot10^{6}$ y
	^{137}Cs	β^- (+γ)	30.1 y
82	**^{206}Pb**		**24.10%**
	^{207}Pb		**22.10%**
	^{208}Pb		**52.40%**
	^{210}Pb	β^- (+γ) α $2\cdot10^{-6}$ %	22.2 y
	^{214}Pb	β^- (+γ)	26.8 min
84	^{210}Po	α (+γ)	138 d
86	^{222}Rn	α (+γ)	3.82 d
88	^{226}Ra	α (+γ)	1600 y
90	**^{232}Th**	α (+γ) **sf 10^{-8} %**	**100%** **$1.41\cdot10^{10}$ y**
92	^{233}U	α (+γ)	$1.59\cdot10^{5}$ y
	^{234}U	α (+γ) **sf $2\cdot10^{-9}$ %**	**0.0054%** **$2.46\cdot10^{5}$ y**
	^{235}U	α (+γ) **sf $7\cdot10^{-9}$ %**	**0.72%** **$7.04\cdot10^{8}$ y**
	^{236}U	α (+γ) sf $9\cdot10^{-8}$ %	$2.34\cdot10^{7}$ y
	^{237}U	β^- (+γ)	6.75 d
	^{238}U	α (+γ) **sf $5\cdot10^{-5}$ %**	**99.27%** **$4.47\cdot10^{9}$ y**
	^{239}U	β^- (+γ)	23.5 min
93	^{236}Np	β^+ 87% (+γ) β^- 13%	$1.54\cdot10^{5}$ y
	^{237}Np	α (+γ)	$2.14\cdot10^{6}$ y
	^{239}Np	β^- (+γ)	2.356 d
94	^{238}Pu	α (+γ) sf $2\cdot10^{-7}$ %	88 y
	^{239}Pu	α (+γ) sf $3\cdot10^{-10}$ %	$2.41\cdot10^{4}$ y
	^{240}Pu	α (+γ) sf $6\cdot10^{-6}$ %	6564 y
	^{241}Pu	β^- (+γ) α 0.003%	14.29 y
	^{242}Pu	α (+γ) sf 0.0006%	$3.75\cdot10^{5}$ y
95	^{241}Am	α (+γ) sf $4\cdot10^{-10}$ %	432 y
	^{243}Am	α (+γ) sf $4\cdot10^{-9}$ %	7370 y
96	^{242}Cm	α (+γ) sf $6\cdot10^{-6}$ %	163 d
	^{244}Cm	α (+γ) sf 0.0001%	18.1 y
98	^{252}Cf	α 96.9% (+γ) sf 3.1%	2.645 y

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