KINS-IAEA Basic Professional Training Course on Nuclear Safety

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Reactor Physics

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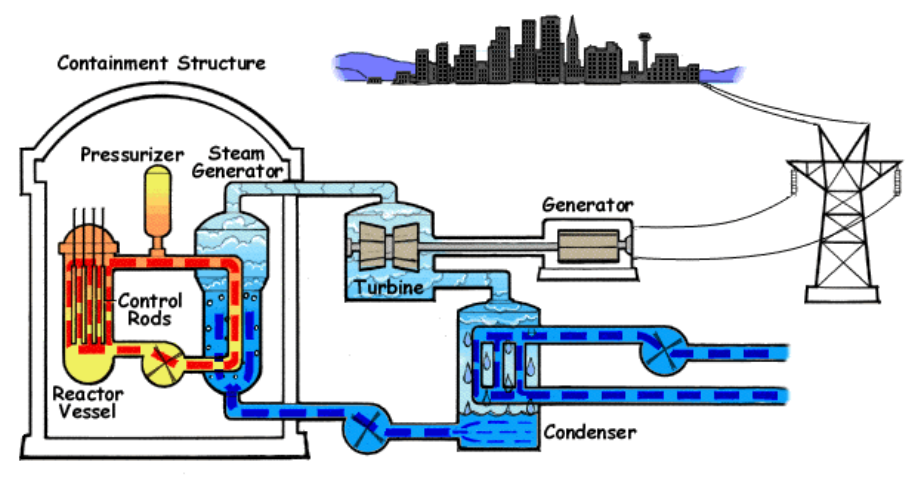
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I. Nuclear Reactors
II. Nuclear Reactions
III. Neutron Transport and Diffusion
IV. Time Dependent Reactor
V. Reactor Safety

I. Nuclear Reactors

❑ Reactor Types

➢ Pressurized Water Reactor



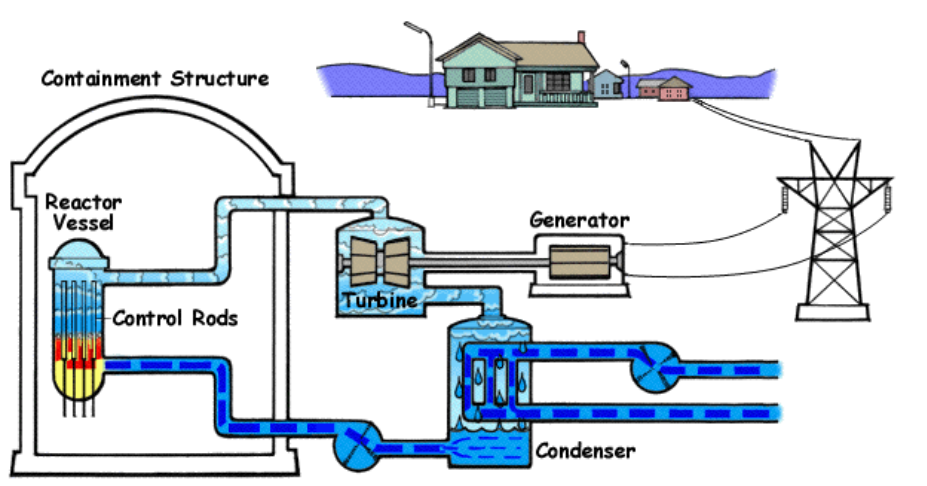
➢ Typical Design Values

	Pressure, MPa	Inlet Temperature, ˚C	Outlet Temperature, ˚C
Primary side	15.5	315 (liquid water)	275 (liquid water)
Secondary side	6.2	220 (liquid water)	275 (saturated steam)

I. Nuclear Reactors

❑ Reactor Types

➢ Boiling Water Reactor



➢ Typical Design Values

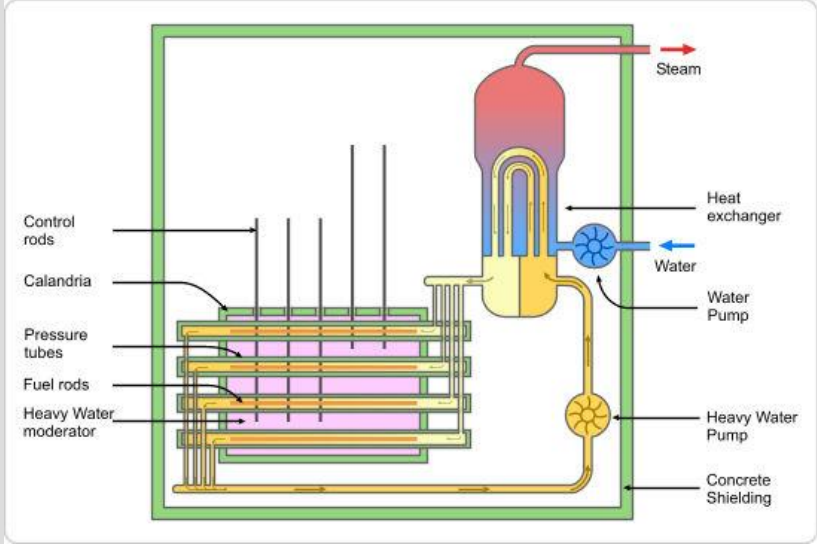
	Pressure, MPa	Inlet Temperature, ˚C	Outlet Temperature, ˚C
Primary side	7	215 (liquid water)	290 (Steam)

I. Nuclear Reactors

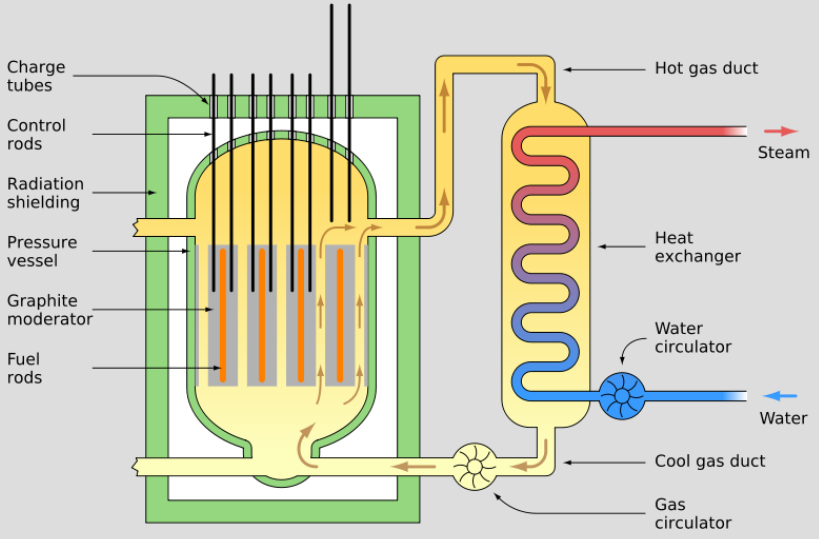
❑ Reactor Types

➢ Pressurized Heavy Water Reactor

- Primary Coolant, Moderator : D_2O
- Secondary Coolant : H_2O



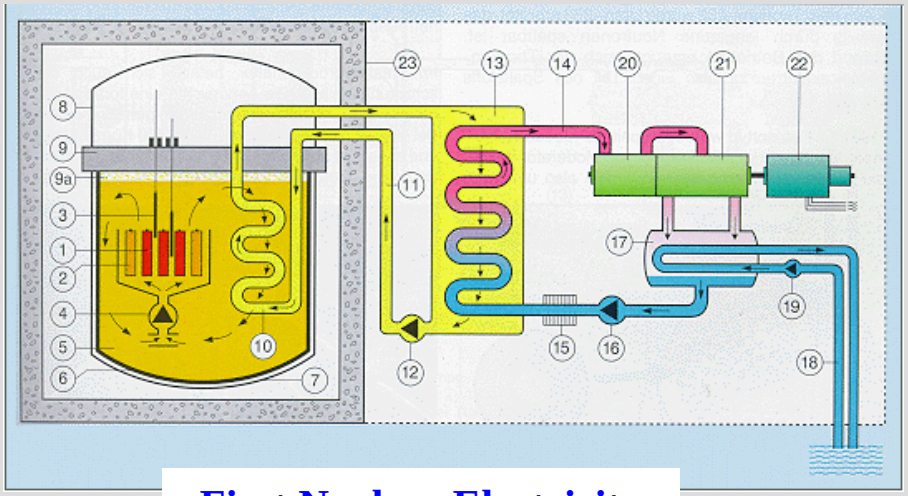
➢ Gas Cooled Reactor

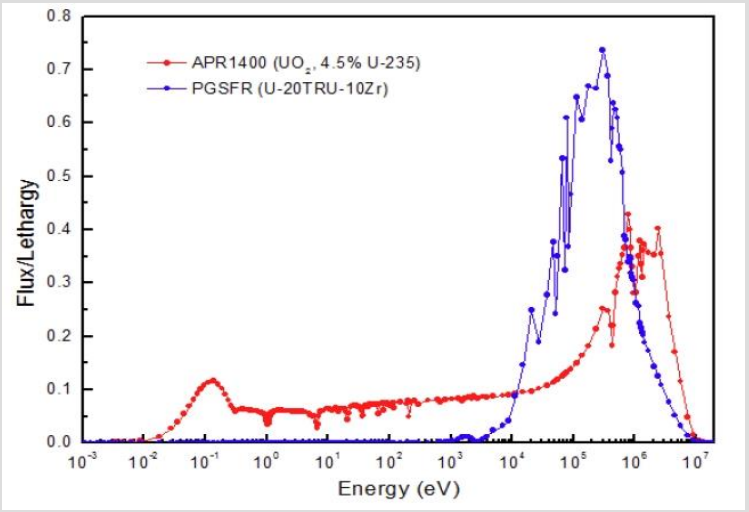


I. Nuclear Reactors

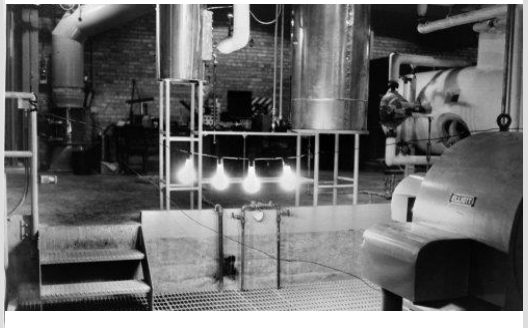
❑ Reactor Types

➢ Fast Breeder Reactor





First Nuclear Electricity



EBR-1, December 20, 1951

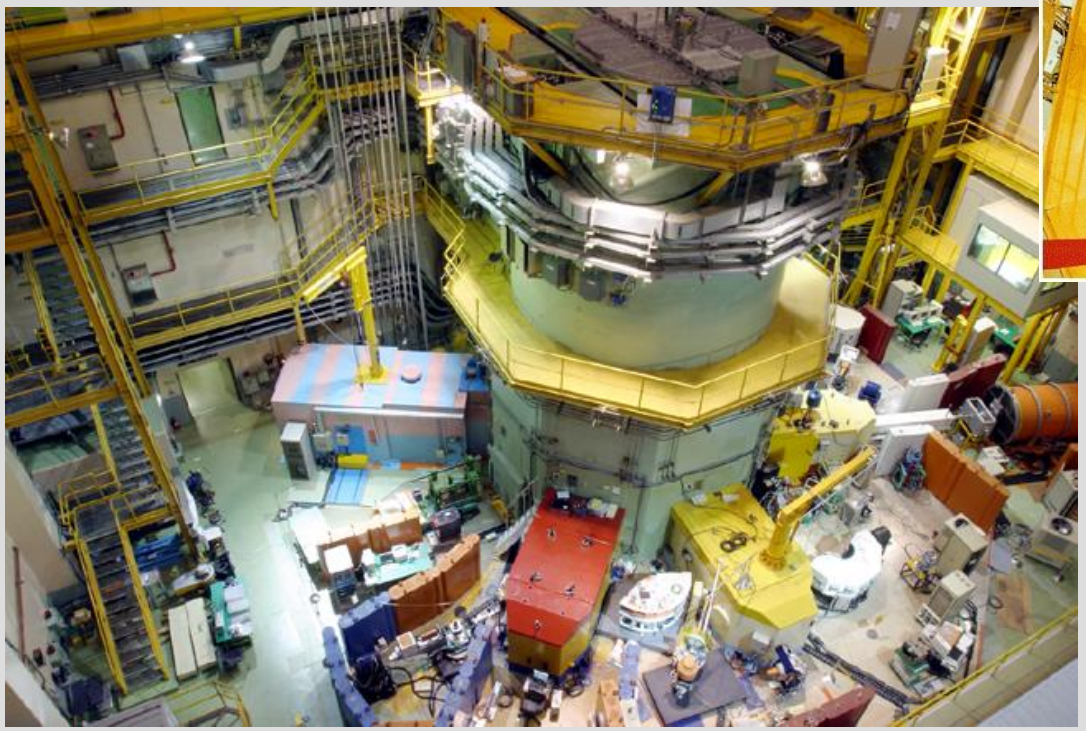
Na and Water Coolants

	Na	H_2O
Melting temperature, ℃	97.8	0
Boiling temperature , ℃	883	100
Density, kg/m^3	880	713
Thermal conductivity, W/mK	76	0.54
Viscosity	0.34	0.1
Specific heat, J/kgK	1,300	5,600
Heat capacity, MJ/m^{3}K	1.14	4.00
Merit	Good material compatibility and heat transfer	Low melting T, Abundancy Good material compatibility
Demerit	Na-water reaction Na fire	Low heat transfer High P

I. Nuclear Reactors

- Reactor Types
  - Research Reactor

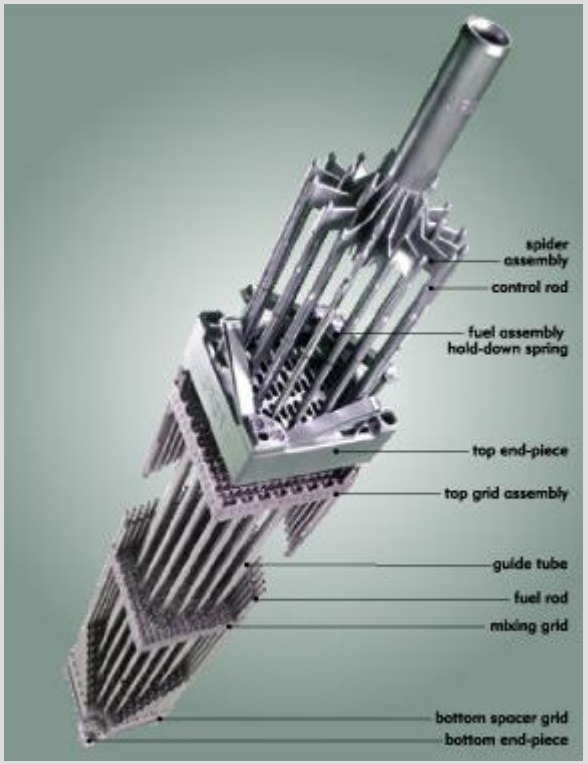




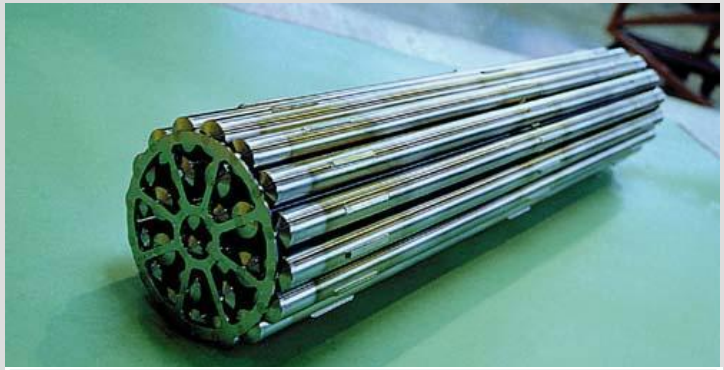
I. Nuclear Reactors

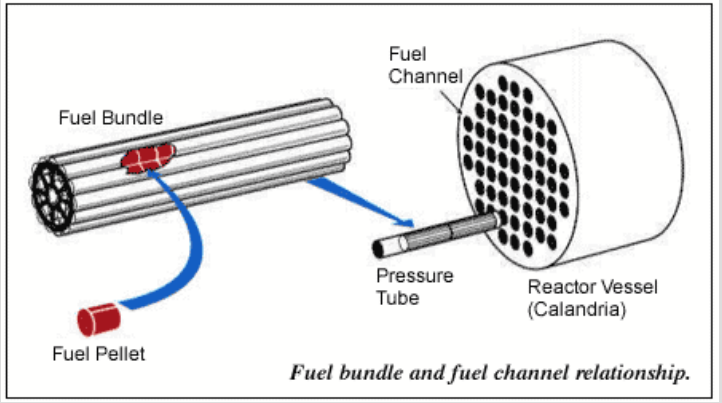
❑ Nuclear Fuel Assemblies

- ➢ PWR fuel assembly



- ➢ CANDU fuel bundle





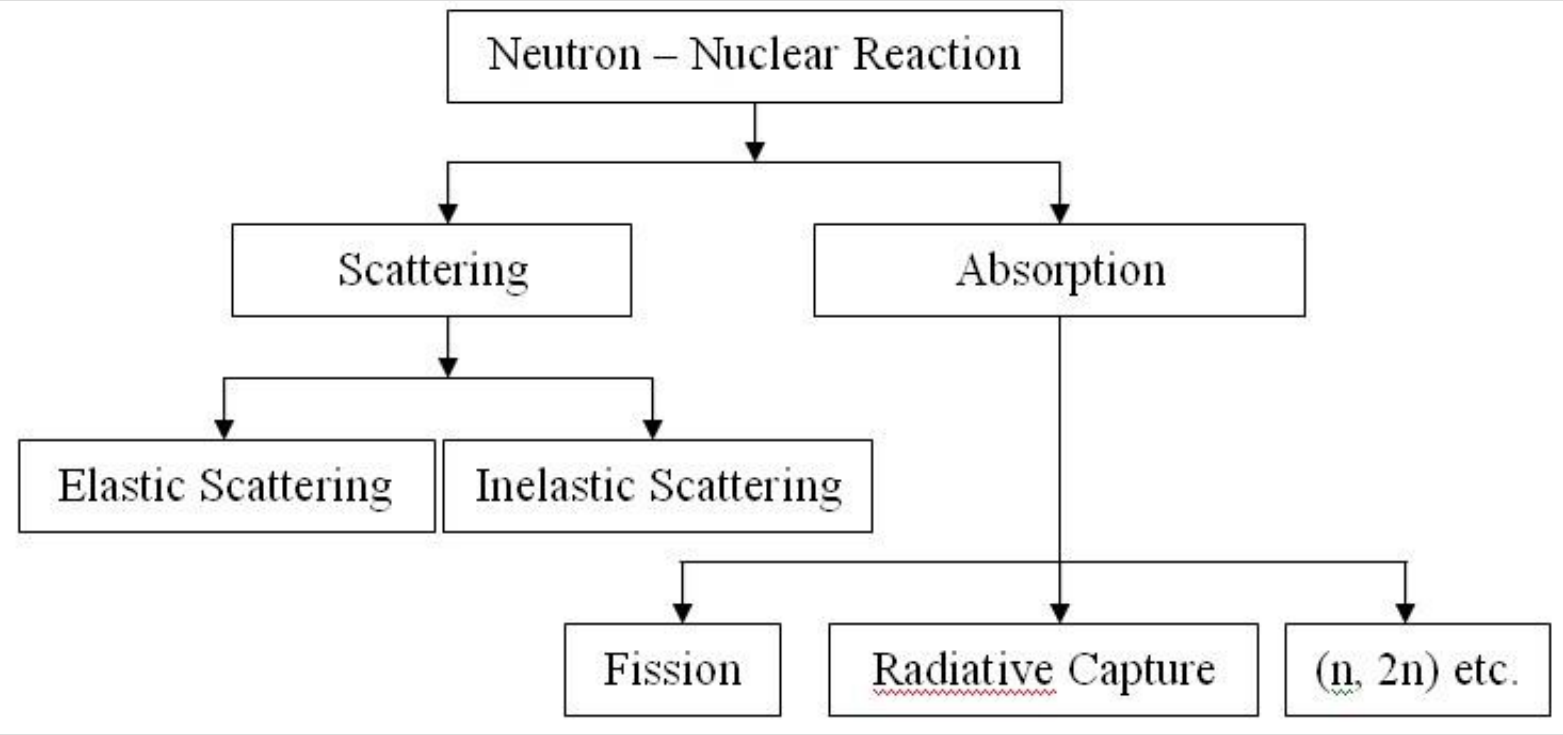
II. Nuclear Reactions

❑ Various Materials in Nuclear Reactor

- There are many different types of nuclear reactors for different purposes
- The structures and components of these reactors are made of various materials (many different nuclides)
- We have to understand the nuclear characteristics of these elements as well as the physical and chemical properties

II. Nuclear Reactions

Neutron induced reactions in Reactors

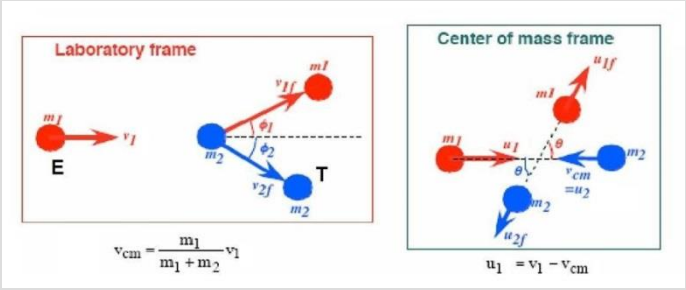


II. Nuclear Reactions

❑ Nuclear Reaction by Neutron

➢ Scattering

- Elastic Scattering



$${}_zX^A + {}_0n^1 \rightarrow {}_zX^A + {}_0n^1$$

- Before and after the scattering, the sum of kinetic energy of the neutron and that of target nucleus remains unchained
- During the scattering, high energy neutron gives its kinetic energy to the target nucleus, thus loses its energy
- The ratio of its kinetic energy before and after the scattering is depending on the mass number *A* of the target nucleus and the scattering angle θ in center of mass frame

$$\frac{E_f}{E_i} = \frac{A^2 + 2A\,Cos\,\theta + 1}{(A+1)^2}, \quad \alpha = \left(\frac{A-1}{A+1}\right)^2$$

II. Nuclear Reactions

- Nuclear reaction by neutron
  - Scattering
    - Inelastic Scattering

      ${}_{Z1}A^{A1} + {}_{0}n^{1} \rightarrow {}_{Z1}A^{A1*} + {}_{0}n^{1}$

      After the scattering, the target nucleus is excited and the sum of kinetic energies of the neutron and target nucleus is smaller than the sum of kinetic energy before the collision
  - Absorption
    - Fission

      ${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236*} \rightarrow {}_{55}Cs^{140} + {}_{37}Rb^{93} + 3{}_{0}n^{1}$

      The excited compound nucleus splits into two fission fragments and a few fission neutrons and releases energy of $\Delta m\, c^2$
    - Capture

      ${}_{92}U^{238} + {}_{0}n^{1} \rightarrow {}_{92}U^{239*} \rightarrow {}_{92}U^{239} + \gamma$

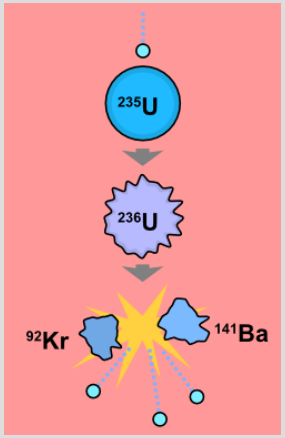
      The neutron is captured by the target nucleus and compound nucleus is stabilized by emitting gamma-ray

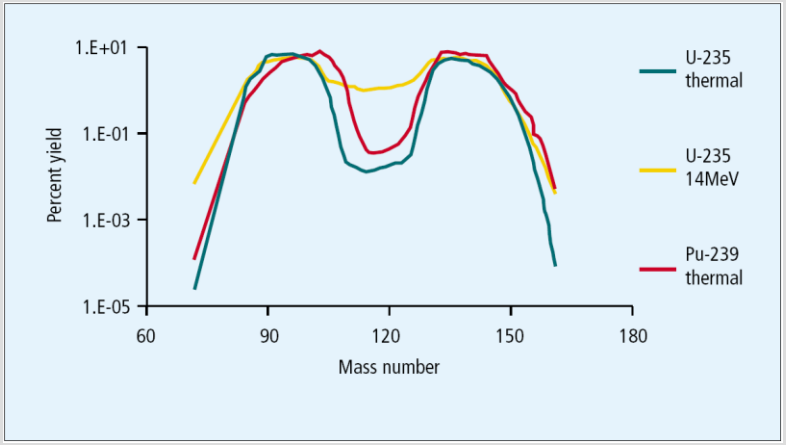
II. Nuclear Reactions

- Fission Reaction by Absorption of Neutron
  - Fission

$$_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236*} \rightarrow {}_{36}Kr^{92} + {}_{56}Ba^{141} + 3{}_{0}n^{1}$$

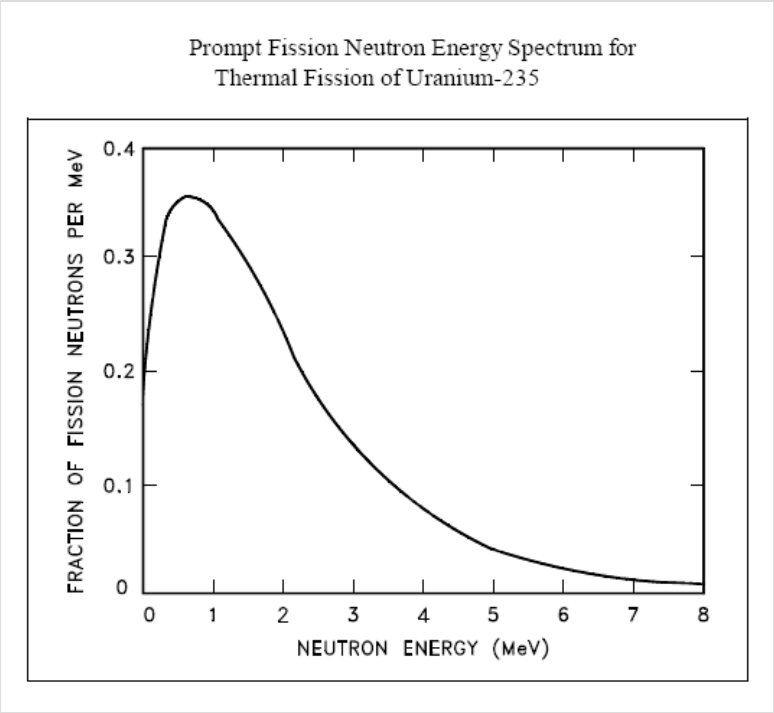
  - The compound nucleus is excited by the neutron absorption and splits into two fission fragments and a few neutrons releasing energy
  - The mass distribution of fission fragments has two peaks showing uneven splitting of the compound nucleus
  - The number of released neutron per fission depends on the fissile nuclide and the incident neutron energy

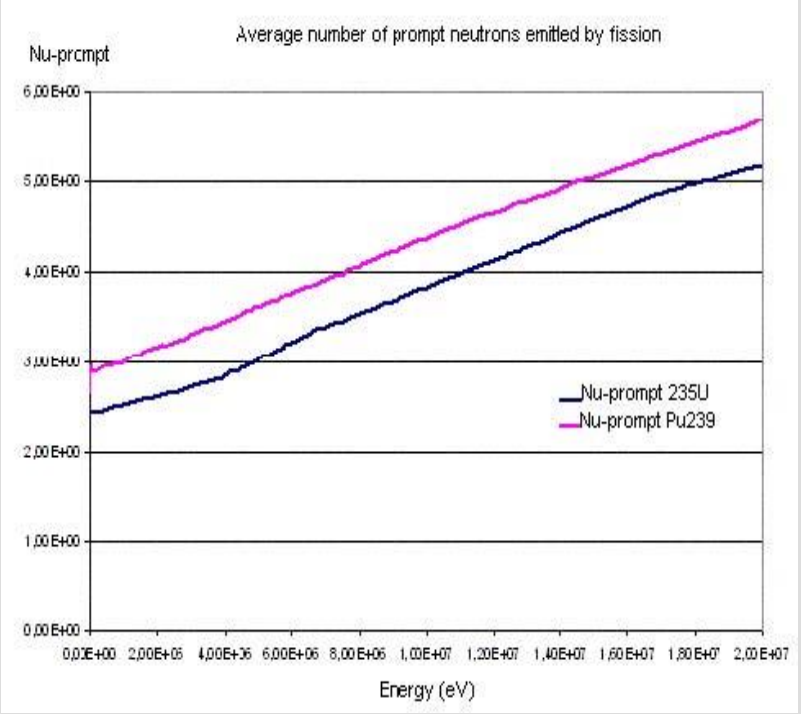




II. Nuclear Reactions

- Fission Reaction by Absorption of Neutron
  - Fission Neutron Energy Spectrum
  - Number of Released Neutron per Fission





II. Nuclear Reactions

- Fission Reaction by Absorption of Neutron
  - Energy Released by Fission

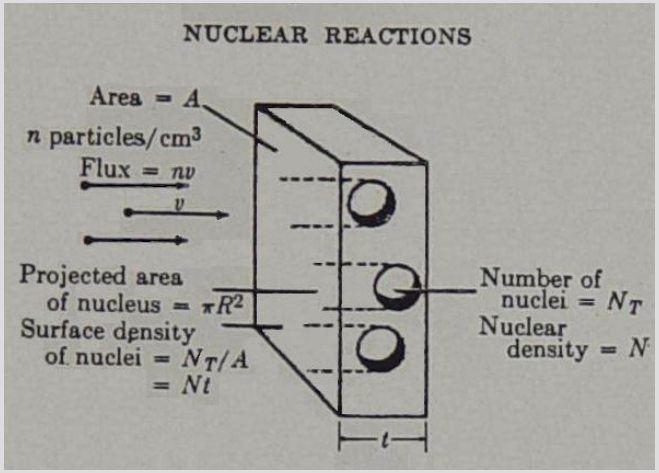
Fission product kinetic energy	168
Neutron kinetic energy	5
Fission gammas (instantaneous)	5
Fission gammas (delayed)	6
Fission product betas	7
Total available as heat	191
Neutrino energy (not available as heat)	11
Total Energy (MeV)	202

Energy distribution per fission induced by thermal neutrons

II. Nuclear Reactions

❑ Reaction Cross Sections

- If the cross section of target nucleus is σ and the number density of nucleus in the target material is N, the probability of reaction for an incident particle per unit thickness is $(\sigma N A t / A) / t = \sigma N = \Sigma$, ($t \quad 0$)
  - σ is called microscopic cross section *(unit of σ : barn $\rightarrow 10^{-24} cm^2$)*
  - Σ is called macroscopic cross section *(unit of Σ : cm^{-1})*
- The average speed of incident particle multiplied by the number density is called **flux** and denoted by $n v = \phi$ *(unit : $cm^{-2} sec^{-1}$)*
  - The reaction rate in the target material is $R = \Sigma \phi$ *(unit : $cm^{-3} sec^{-1}$)*



$$\text{Probability of Reaction for an incident particle} = \frac{\text{Area Shadowed by Nuclei}}{\text{Total Area}} = \frac{\sigma N A t}{A} = \sigma N t$$

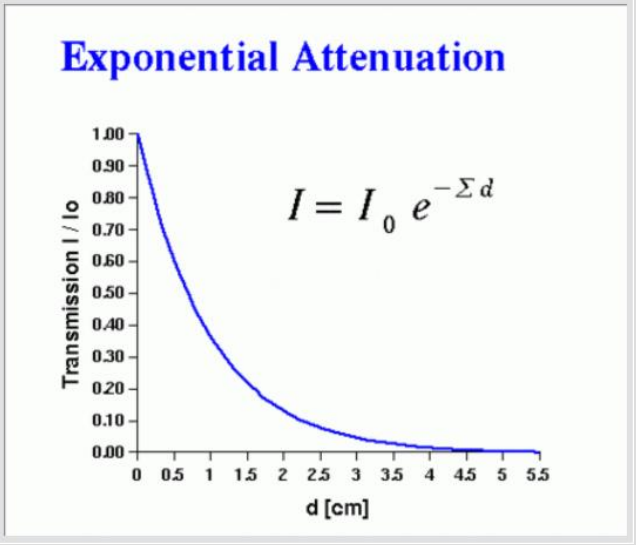
$$\text{Probability of Reaction per unit length} = \frac{\sigma N t}{t} = \sigma N = \Sigma$$

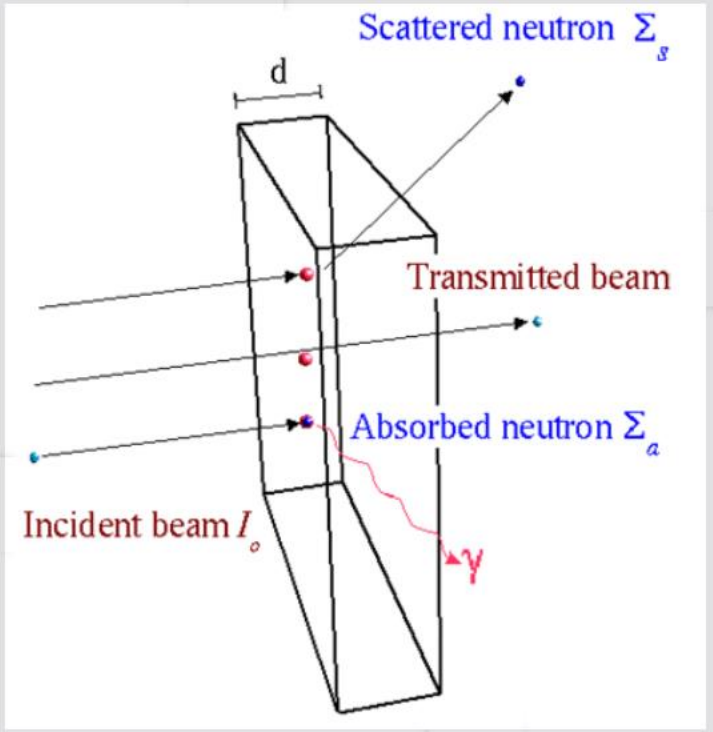
II. Nuclear Reactions

❑ Neutron Attenuation

- ➢ The attenuation of neutron beam by medium is given by the following equation

$$\frac{d\phi}{dx} = -\Sigma\,\phi$$

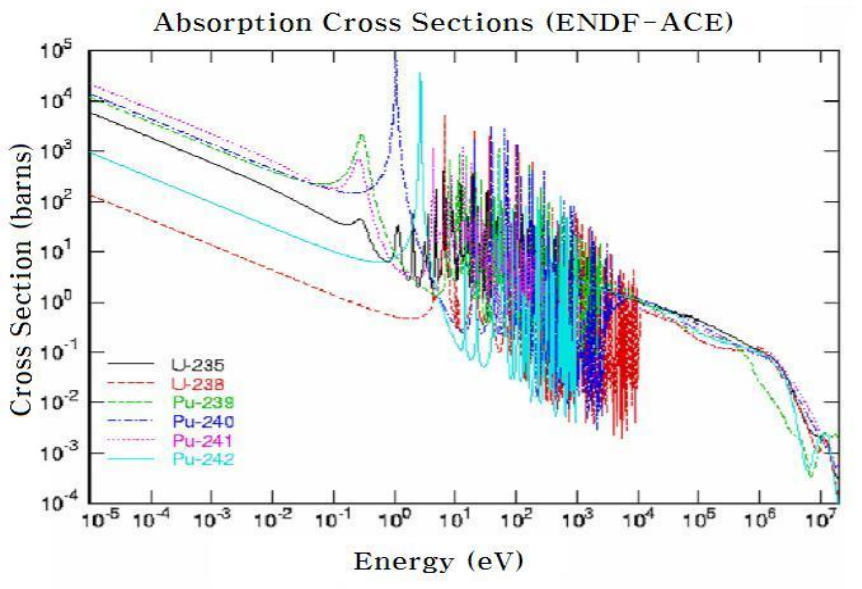


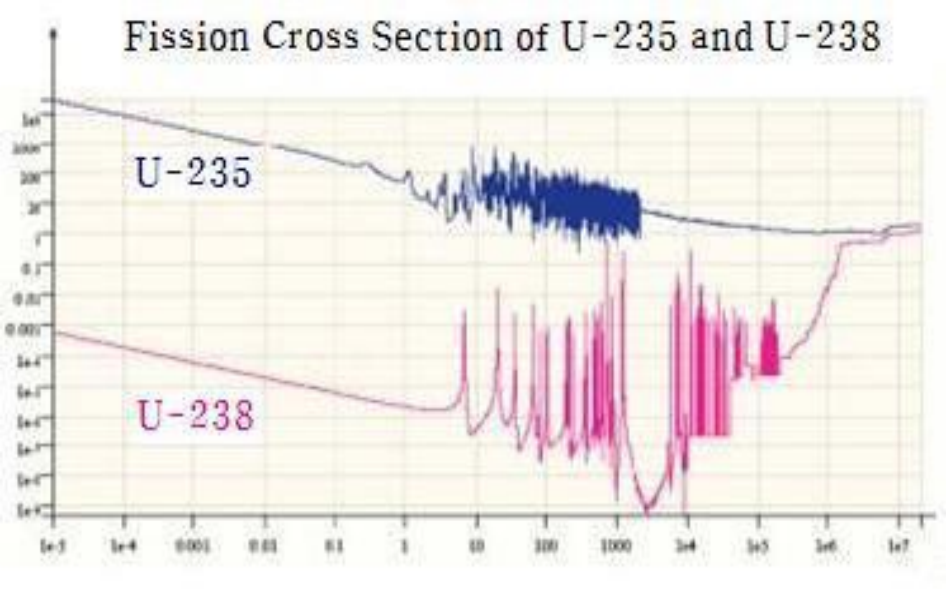


II. Nuclear Reactions

❑ Reaction Cross Sections

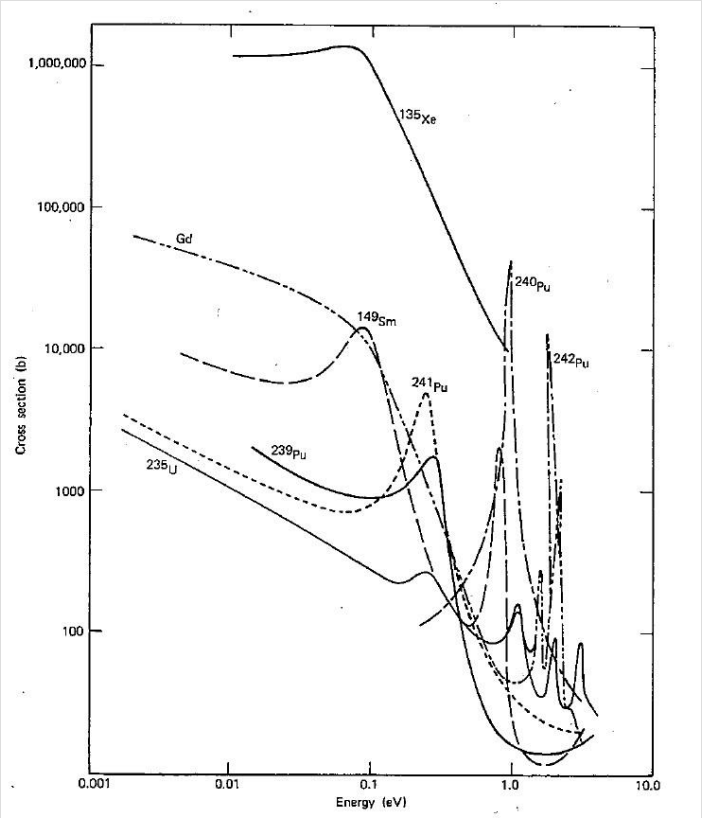
- ➢ Reaction cross sections depend on the target nuclei and the energy of the incident neutron
- ➢ Reaction cross sections are different for different type of reactions

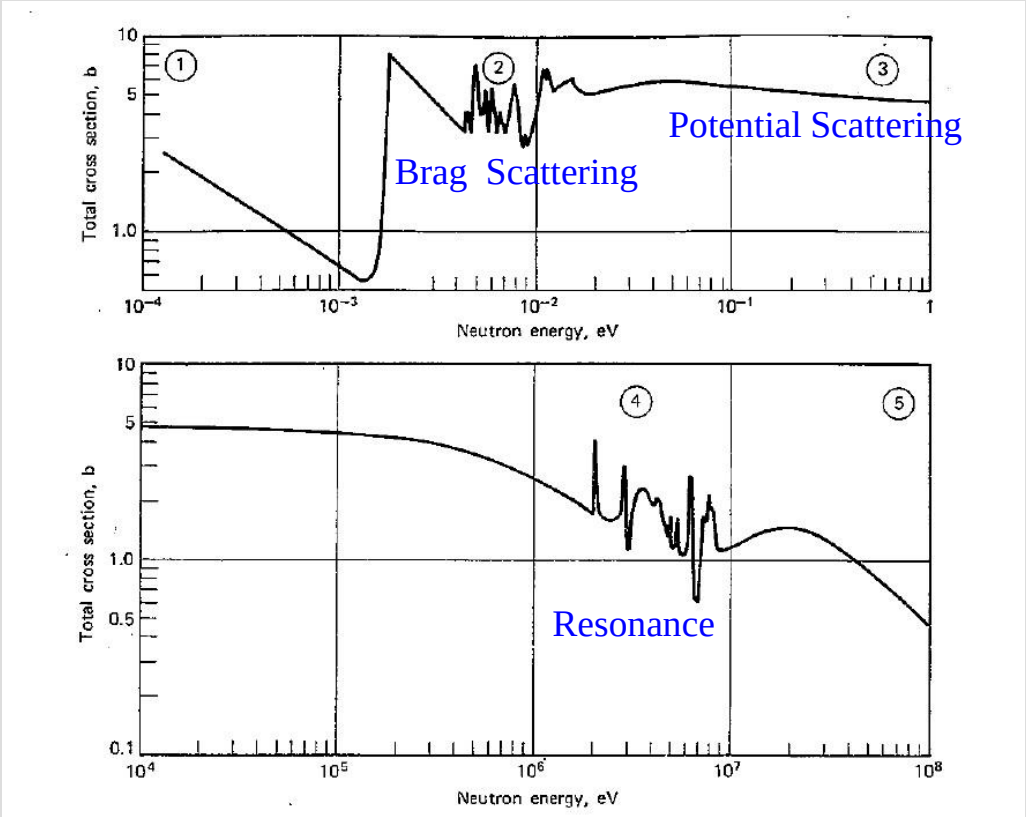




II. Nuclear Reactions

❑Reaction Cross Sections





II. Nuclear Reactions

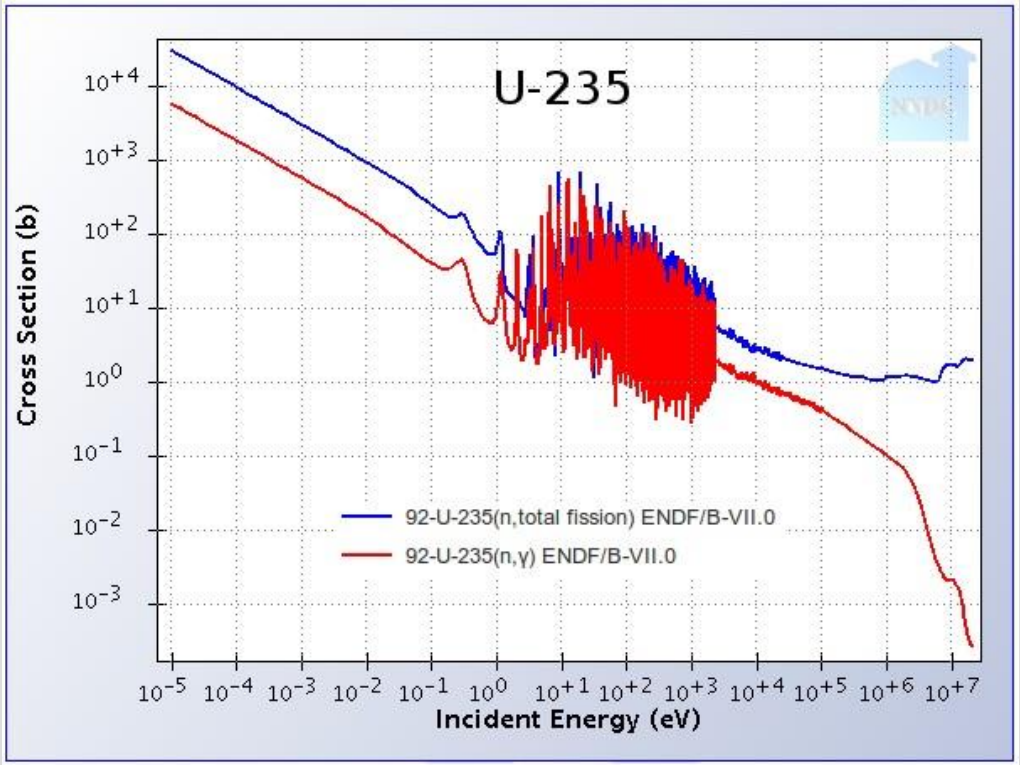
- Typical Reaction Cross Sections of a PWR Core

TABLE 5-2 Number Density and Microscopic Cross Sections

Material	N (1/b-cm)	$\sigma_a(b)$	$\sigma_f(b)$	$\Sigma_a(\text{cm}^{-1})$	$\nu\Sigma_f(\text{cm}^{-1})$
H	2.748×10^{-2}	0.294	0	8.08×10^{-3}	0
O	2.757×10^{-2}	1.78×10^{-4}	0	4.90×10^{-6}	0
Zr	3.694×10^{-3}	0.190	0	7.01×10^{-4}	0
Fe	1.710×10^{-3}	2.33	0	3.99×10^{-3}	0
^{235}U	1.909×10^{-4}	484.0	312.0	9.24×10^{-2}	0.145
^{238}U	6.592×10^{-3}	2.11	0.638	1.39×10^{-2}	1.20×10^{-2}
^{10}B	1.001×10^{-5}	3.41×10^{-3}	0	3.41×10^{-2}	0
				0.1532	0.1570

II. Nuclear Reactions

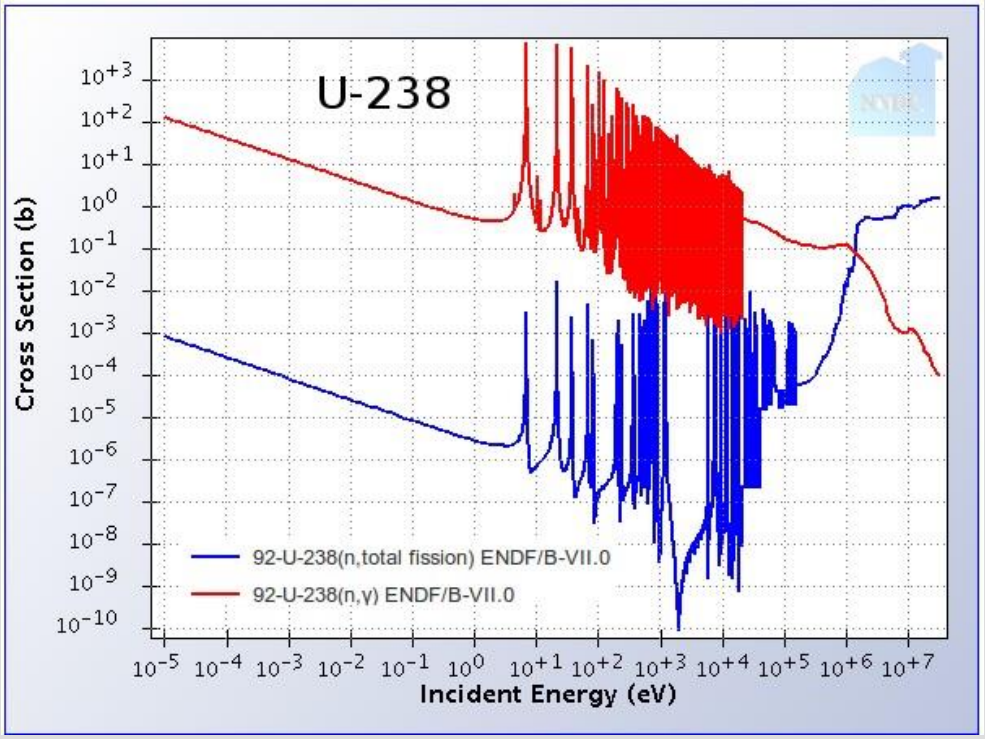
- Fission and Absorption Cross Sections of Uranium-235
  - U-235 is the only naturally occurring fissile isotope



II. Nuclear Reactions

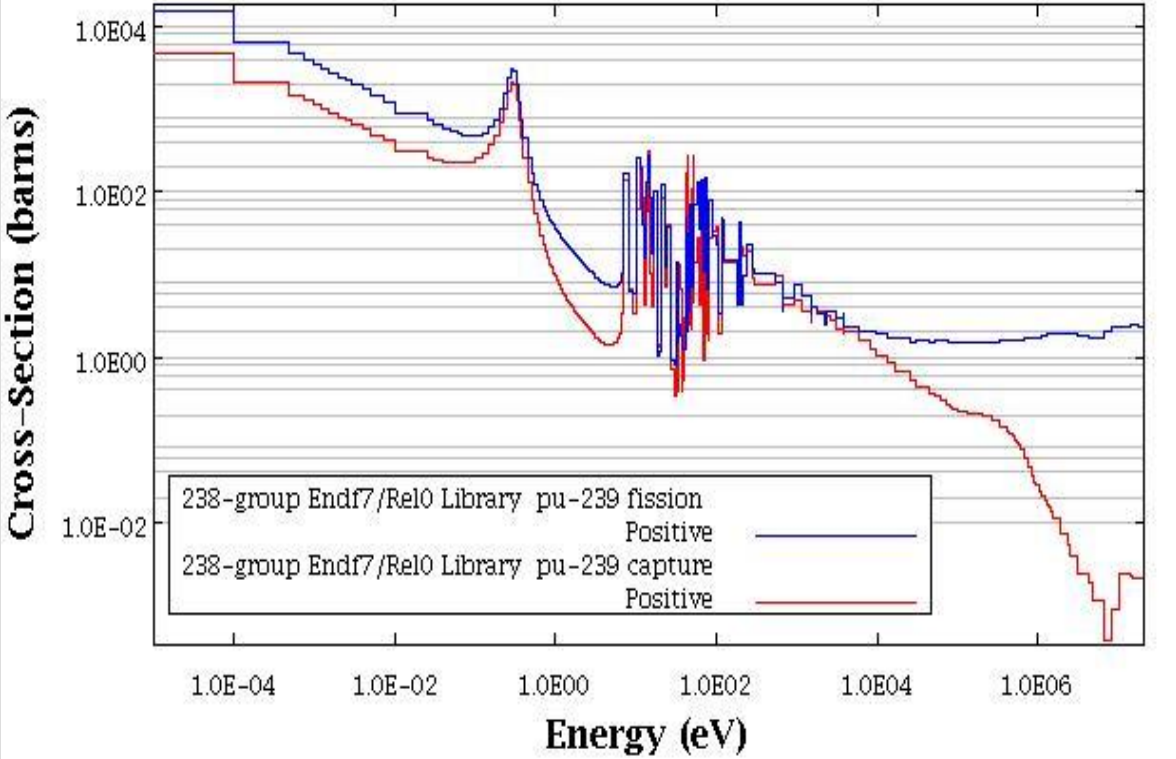
❑ Fission and Absorption Cross Sections of Uranium-238

- ➢U-238 is fissionable by fast neutrons
- ➢Capable of being transmuted to fissile Pu-239 in a nuclear reactor



II. Nuclear Reactions

- Fission and Absorption Cross Sections of Pu-239

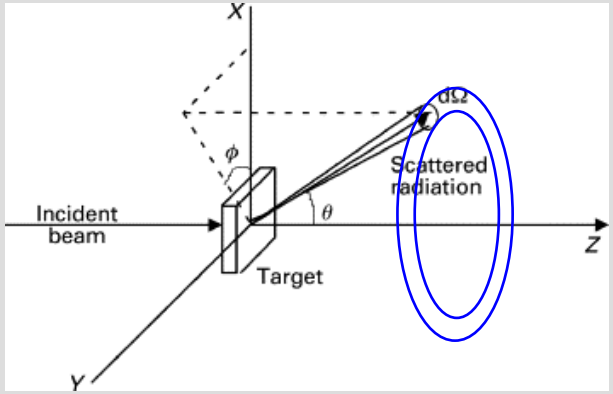


II. Nuclear Reactions

Elastic Collision

In Elastic Collision, the total kinetic energy of the incident neutron and the target nucleus is conserved after collision. The ratio of neutron kinetic energy at L-frame is given below. θ is the scattering angle of neutron at CM-frame

$$\frac{E_f}{E_i} = \frac{A^2 + 2A\,Cos\,\theta + 1}{(A+1)^2}$$



By the head on collision, neutron lose its kinetic energy at maximum

$$\frac{E_f}{E_i} = \frac{(A-1)^2}{(A+1)^2} = \alpha$$

II. Nuclear Reactions

Average Neutron Energy after Scattering

Fission neutron experience repeated collisions with medium. Neutron Kinetic energy after ***n*** collisions can be written as the following equations.

$$E_f = E_i \; r_1 \; r_1 \; r_1 \ldots r_n$$

$$\ln\left(\frac{E_i}{E_f}\right) = \ln\left(\frac{1}{r_1 \; r_2 \; r_3 \ldots r_n}\right) = \ln\left(\frac{1}{r_1}\right) + \ln\left(\frac{1}{r_2}\right) + \cdots + \ln\left(\frac{1}{r_n}\right)$$

$$= \sum_{i=1}^{n} \ln\left(\frac{1}{r_i}\right) = n \cdot \overline{\ln\left(\frac{1}{r_i}\right)} = n \; \overline{\ln\left[\frac{(1+A)^2}{1+2A\,Cos(\theta)+A^2}\right]} = n \; \xi$$

$$\xi = 1 - \frac{(A-1)^2}{2A} \ln\left(\frac{A+1}{A-1}\right) \quad \text{for Isotropic Scattering}$$

II. Nuclear Reactions

❑ Average Number of Collision

For a fission neutron to become a thermal neutron, the average number of collision n can be calculated as follows

For the graphite medium, the average number of collision is 115.25 for 2 MeV neutron to become 0.025 eV

$$n = \frac{1}{\xi}\ln\left(\frac{E_i}{E_f}\right) \approx \frac{3A+2}{6}\ln\left(\frac{E_i}{E_f}\right)$$

$$= \frac{36+2}{6}\ln(80,000,000) = 115.25$$

III. Neutron Transport and Diffusion

1. Neutron Transport

- Boltzmann Transport Equation

$$\frac{\partial}{\partial t} n(r, E, \Omega, t) = \frac{1}{v} \frac{\partial}{\partial t} \varphi(r, E, \Omega, t)$$

$$= \iint \Sigma_s(r, E' \to E, \Omega' \to \Omega) \ \varphi(r, E', \Omega', t) \, dE' \, d\Omega'$$

$$+ \frac{\chi(E)}{4\pi} \iint \nu\Sigma_f(r, E') \ \varphi(r, E', \Omega', t) \, dE' \, d\Omega'$$

$$- \Omega \cdot \nabla \varphi(r, E, \Omega, t) - \Sigma_t(r, E) \ \varphi(r, E, \Omega, t) \quad -----(1)$$

III. Neutron Transport and Diffusion

2. Neutron Diffusion

- Diffusion Equation

Integration of Boltzmann Eq. (1) by 4π solid angle results

$$\frac{1}{v}\frac{\partial \phi(r,E,t)}{\partial t} = -\nabla \cdot J - \Sigma_t(r,E)\ \phi(r,E,t) +$$

$$\int_0^\infty \Sigma_s(r,E' \to E)\ \phi(r,E',t)\,dE' + \chi(E)\int_0^\infty \nu\Sigma_f(r,E')\ \phi(r,E',t)\,dE' \quad --(2)$$

$$\text{where,} \quad J = \int_{4\pi} \Omega\,\varphi(r,E,\Omega,t)\,d\Omega$$

As s a result of integration, direction vector, **Ω,** disappeared but a new unknown ***J*** appeared

III. Neutron Transport and Diffusion

If the angular dependence is not strong, angular flux can be approximated by first order function of **Ω**

$$\varphi(r,E,\Omega) \cong \frac{1}{4\pi}\phi(r,E) + \frac{3}{4\pi}J(r,E)\cdot\Omega \quad -----(3)$$

Multiplying the Boltzmann equation by **Ω,** and integration over 4π solid angle results in the following equation

$$\frac{1}{v}\frac{\partial J(r,E,t)}{\partial t} + \frac{1}{3}\nabla\phi(r,E,t) + \Sigma_{tr}(r,E)\ J(r,E,t) = 0 \quad ---(4)$$

$$J(r,E,t) = -\frac{1}{3\Sigma_{tr}(r)}\nabla\phi(r,E,t) = -D(r)\nabla\phi(r,E,t) \quad ---(5)$$

III. Neutron Transport and Diffusion

By substitution of ***J*** of Eq. (2) into Eq. (5) results in the following diffusion equation

$$\frac{1}{v}\frac{\partial \phi(r,E,t)}{\partial t} = \nabla\cdot[D(r,E)\nabla\phi(r,E,t)] - \Sigma_t(r,E)\ \phi(r,E,t) +$$

$$\int_0^\infty \Sigma_s(r,E'\to E)\ \phi(r,E',t)\,dE' + \chi(E)\int_0^\infty \nu\Sigma_f(r,E')\ \phi(r,E',t)\,dE' \quad --(6)$$

where, $D(r,E) = \dfrac{1}{3\,\Sigma_{tr}(r,E)} = \dfrac{1}{3[\Sigma_t(r,E) - \bar{\mu}\,\Sigma_s(r,E)]}$

III. Neutron Transport and Diffusion

- Equation (6) can be written for a large number of neutron energy intervals to formulate the **Multi-group Diffusion Equation**

$$\frac{1}{v}\frac{\partial \phi_g(r,E,t)}{\partial t} = \nabla\cdot[D_g\nabla\phi_g(r)] - \Sigma_{tg}\phi_g(r) + \sum_{g'=1}^{N}[\Sigma_{sg'\to g} + \chi_g \nu\Sigma_{fg'}]\phi_{g'}(r) \quad -(7)$$

$$\phi_g(r) \equiv \int_{E_g}^{E_{g-1}} \phi_g(r,E)dE \ , \qquad \Sigma_{xg} \equiv \frac{\int_{E_g}^{E_{g-1}} \Sigma_x(E)\,\phi(E)\,dE}{\int_{E_g}^{E_{g-1}} \phi(E)\,dE}$$

- For commercial nuclear reactor, 2 Group diffusion equation is the most commonly used
- Due to the advances in computing power, direct transport calculation is possible

III. Neutron Transport and Diffusion

Dividing the neutron production term in Eq. (7) by the multiplication factor, we can obtain the neutron valance between production and consumption, and thus, left hand time derivative term vanishes

$$\frac{1}{v}\frac{\partial \phi(r,E,t)}{\partial t} = \nabla\cdot[D_g \nabla\phi_g(r)] - \Sigma_{tg}\phi_g(r) + \sum_{g'=1}^{N}[\Sigma_{sg'\to g} + \chi_g \nu\Sigma_{fg'}]\phi_{g'}(r)$$

$$\Downarrow$$

$$0 = \nabla\cdot[D_g \nabla\phi_g(r)] - \Sigma_{tg}\phi_g(r) + \sum_{g'=1}^{N}[\Sigma_{sg'\to g} + \frac{\chi_g}{k_{eff}}\nu\Sigma_{fg'}]\phi_{g'}(r)$$

III. Neutron Transport and Diffusion

❑ One-Group Diffusion Equation

- ➢ Even though the behavior of the cross sections and diffusion coefficients are strongly dependent on the neutron energy, we can choose the entire energy interval $E = 0 \sim \infty$ as a single group with the averaged parameters that preserve the meaningful outcomes such as reaction rates

$$\frac{1}{v}\frac{\partial \varphi(r,t)}{\partial t} = \nabla \cdot (D(r) \nabla \varphi(r,t)) - \Sigma_a(r)\, \varphi(r,t) + \nu \Sigma_f(r)\, \varphi(r,t) \quad --(1)$$

- ➢ If the reactor is critical, i.e., k_{eff} =1, and homogeneous the time derivative terms in Eq.(1) vanish and the neutron balance equation becomes

$$\nabla \cdot (D(r) \nabla \phi(r,t)) = -[\, \nu \Sigma_f(r) - \Sigma_a(r)\,] \phi(r,t) \,, \rightarrow \nabla^2 \phi(r,t) = -B_m^{\,2}\, \phi(r,t) \quad --(2)$$

$$\textit{where} \quad B_m^{\,2} = \frac{\nu \Sigma_f - \Sigma_a}{D}$$

III. Neutron Transport and Diffusion

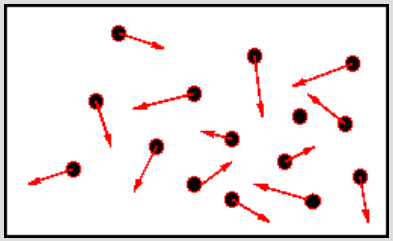
Neutron Flux, Current and Diffusion

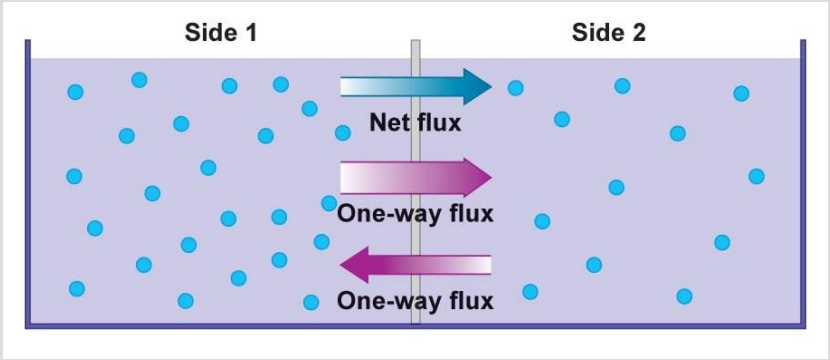
- Diffusion is a Random Walk process
- The rate of neutron flow (net neutron current) is proportional to the flux gradient.

$$\vec{J} = \sum_{i}^{\text{unit volume}} \vec{v}_i = n\bar{\vec{v}}$$

$$\phi = \sum_{i}^{\text{unit volume}} v_i = n\bar{v}$$

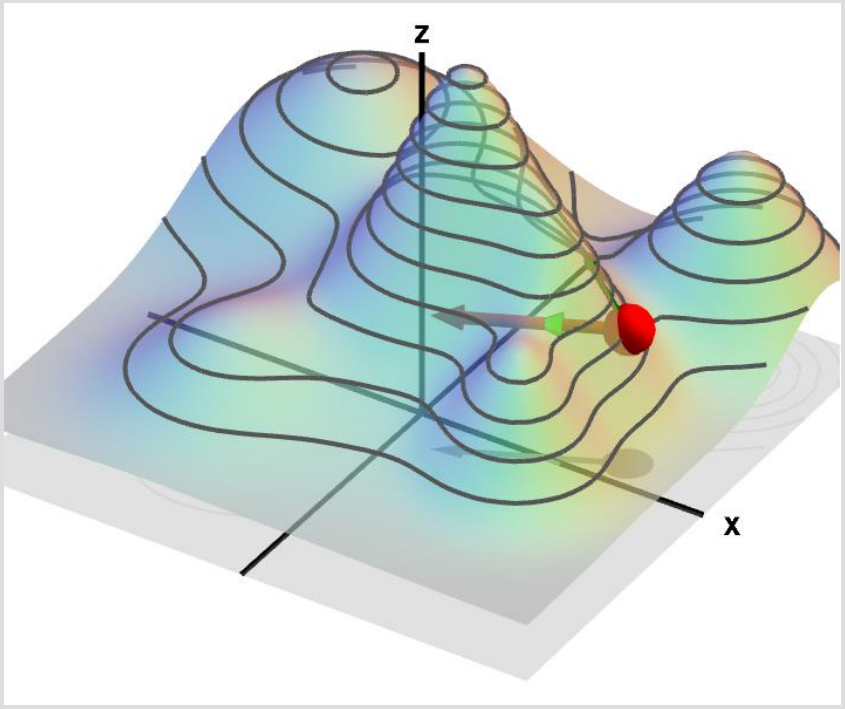
$$\vec{J} = -D\,\nabla\phi$$

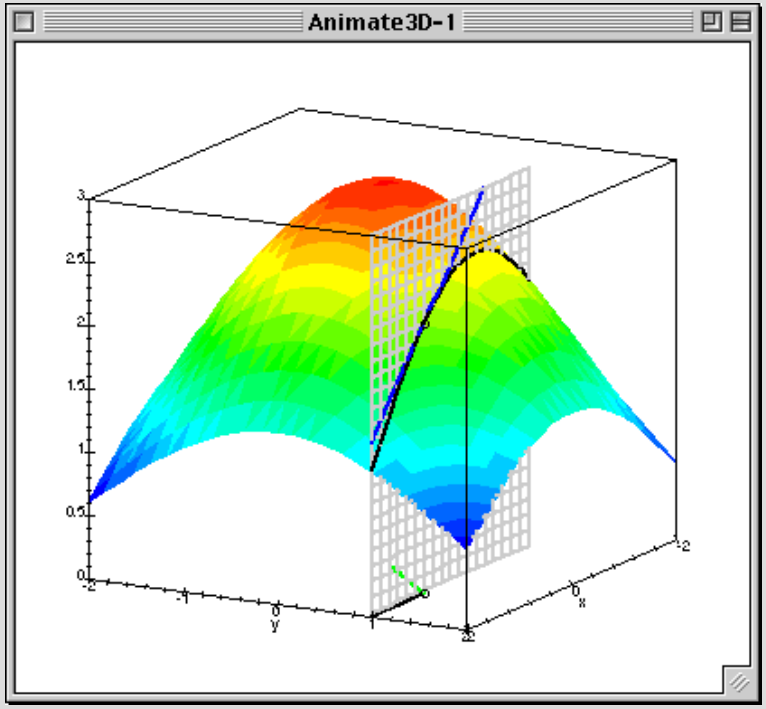




III. Neutron Transport and Diffusion

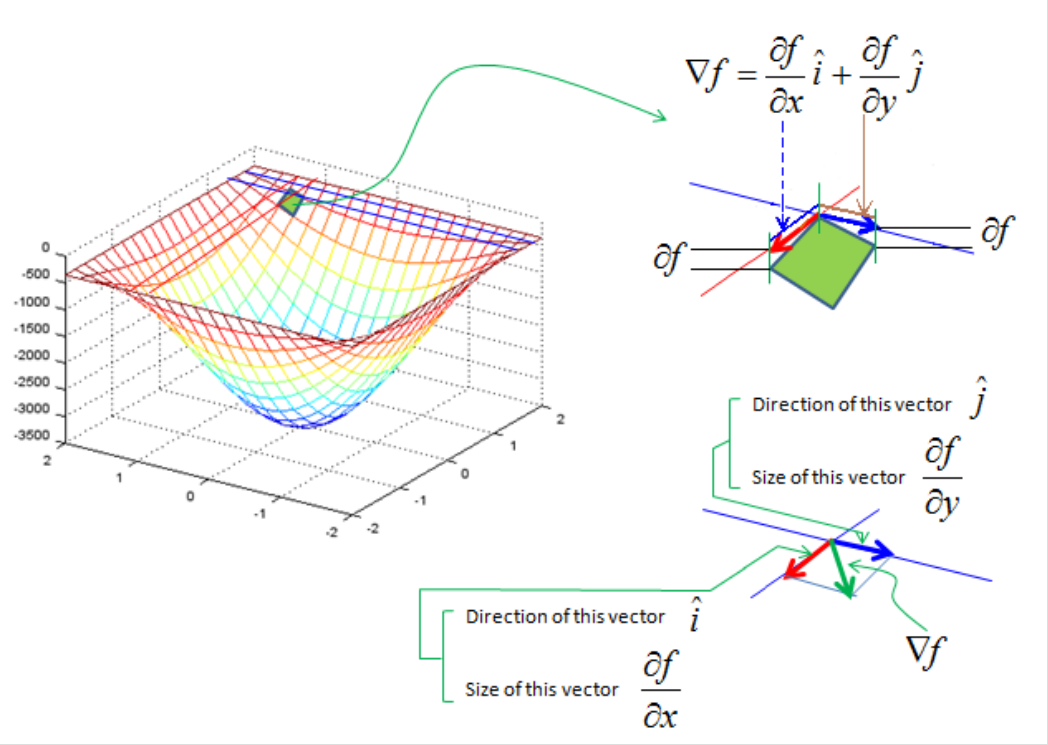
- Gradient of Scalar Function
  - The Gradient of a scalar is the rate of change of the that scalar quantity per unit length





III. Neutron Transport and Diffusion

- Gradient of Scalar Function
  - The Gradient of a scalar is a vector, component of which is the rate of change of the scalar quantity in that direction
  - In two dimension



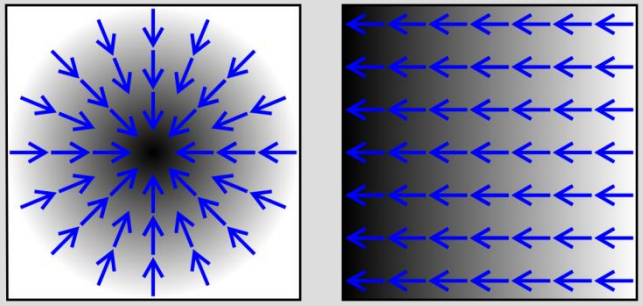
III. Neutron Transport and Diffusion

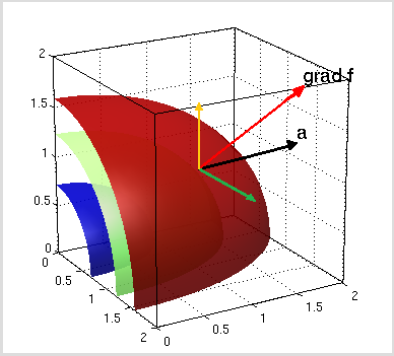
- Gradient of Scalar Function
  - In three dimension, the gradient operator, ∇, becomes

$$\nabla = \vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\ ,\ \textit{Cartesian Coordinate System}$$

$$\nabla = \vec{u}_r\frac{\partial}{\partial r} + \vec{u}_\varphi\frac{1}{r}\frac{\partial}{\partial \varphi} + \vec{u}_z\frac{\partial}{\partial z}\ ,\ \textit{Cylindrical Coordinate System}$$

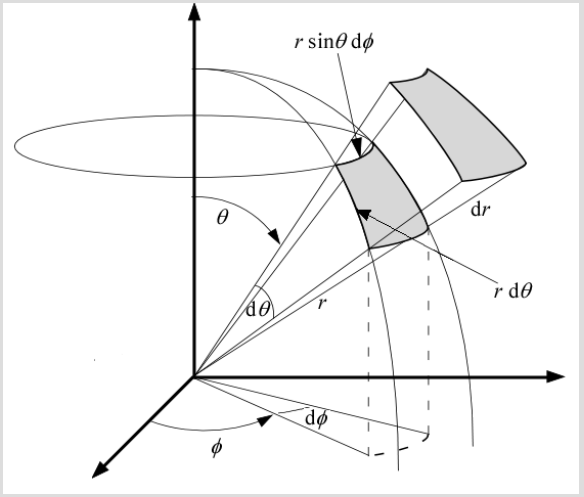
$$\nabla = \vec{u}_r\frac{\partial}{\partial r} + \vec{u}_\theta\frac{1}{r}\frac{\partial}{\partial \theta} + \vec{u}_\varphi\frac{1}{r\,Sin(\theta)}\frac{\partial}{\partial \varphi}\ ,\ \textit{Spherical Coordinate System}$$





III. Neutron Transport and Diffusion

- Orthogonal Curve Linear Coordinate Systems
  - For spherical coordinate system



$$x = r\ Sin(\theta)\ Cos(\varphi) \qquad h_1 = 1$$
$$y = r\ Sin(\theta)\ Sin(\varphi) \qquad h_2 = r$$
$$z = r\ Cos(\theta) \qquad h_3 = r\ Sin(\theta)$$

$$dl_\theta = \sqrt{dx^2 + dy^2 + dz^2} = \sqrt{\left(\frac{\partial x}{\partial \theta}\right)^2 + \left(\frac{\partial y}{\partial \theta}\right)^2 + \left(\frac{\partial z}{\partial \theta}\right)^2}\ d\theta$$

$$= \sqrt{\left(r\,Cos(\theta)\,Cos(\varphi)\right)^2 + \left(r\,Cos(\theta)\,Sin(\varphi)\right)^2 + \left(r\ Sin(\theta)\right)^2}\ d\theta = r\ d\theta$$

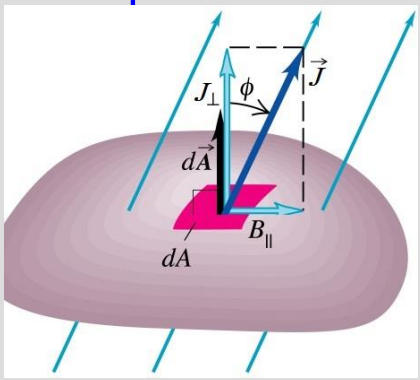
By the same argument, $dl_r = dr$ *and* $dl_\varphi = r\ Sin(\theta)\ d\varphi$

$$dx\,\vec{i} + dy\,\vec{j} + dz\,\vec{k} = 1\,dr\,\vec{u}_r + r\,d\theta\,\vec{u}_\theta + r\ Sin(\theta)\,d\varphi\,\vec{u}_\varphi$$

III. Neutron Transport and Diffusion

Divergence of Neutron Current

- The number of neutron which pass through a given surface per unit time per unit area is shown in the following figure



The rate of outgoing neutron through the surface dA is $= \vec{J} \cdot \vec{n}\, dA$

where $\vec{n}$ *is outgoing unit vector perpendicular to the surface*

- The total number of outgoing neutrons through a surface surrounding a volume is given by the following equation

The rate of outgoing neutron

$$= \iint \vec{J} \cdot \vec{n}\, dA = \iiint \nabla \cdot \vec{J}\, dv$$

III. Neutron Transport and Diffusion

Divergence of a Vector

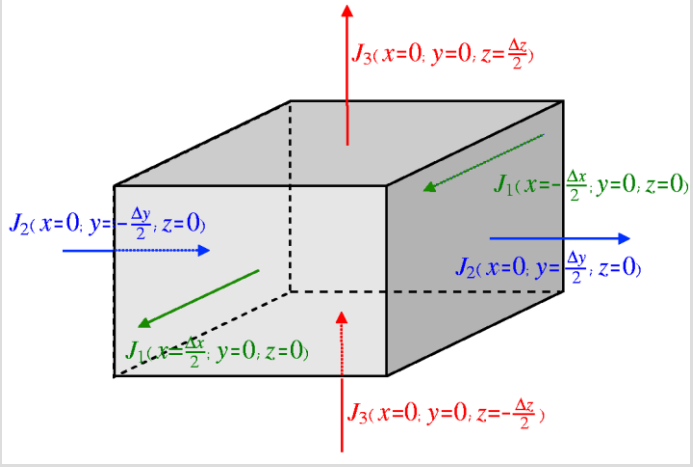
- The sum of outgoing neutrons from the front surface and the back surface of the small box in the figure below is

$$= \Delta y\,\Delta z\ [\,J_x(\frac{\Delta x}{2},0,0) - J_x(\frac{-\Delta x}{2},0,0)] \ = \ \Delta y\,\Delta z\,\Delta x\frac{\partial J_x(0,0,0)}{\partial x}$$

- By the same argument, the net outgoing neutrons from the surface of y and z directions are

$$\Delta x\,\Delta z\ [\,J_y(\frac{\Delta y}{2},0,0) - J_y(\frac{-\Delta y}{2},0,0)] \ = \ \Delta z\,\Delta x\,\Delta y\frac{\partial J_y(0,0,0)}{\partial y}$$

$$\Delta x\,\Delta y\ [\,J_z(\frac{\Delta z}{2},0,0) - J_z(\frac{-\Delta z}{2},0,0)] \ = \ \Delta x\,\Delta y\,\Delta z\frac{\partial J_z(0,0,0)}{\partial z}$$



III. Neutron Transport and Diffusion

- Divergence of a Vector
  - The total number of outgoing neutrons from the box at origin is

$$\Delta x\,\Delta y\,\Delta z\;\left[\frac{\partial J_x(0,0,0)}{\partial x}+\frac{\partial J_y(0,0,0)}{\partial y}+\frac{\partial J_z(0,0,0)}{\partial z}\right]=\Delta V\;\left[\nabla\cdot\vec{J}\right]$$

  - For arbitrary point in space, the total number of outgoing neutrons per unit volume per second is given by the divergence operation on J

$$\nabla\cdot\vec{J}(x,y,z)=\frac{\partial J_x(x,y,z)}{\partial x}+\frac{\partial J_y(x,y,z)}{\partial y}+\frac{\partial J_z(x,y,z)}{\partial z}$$

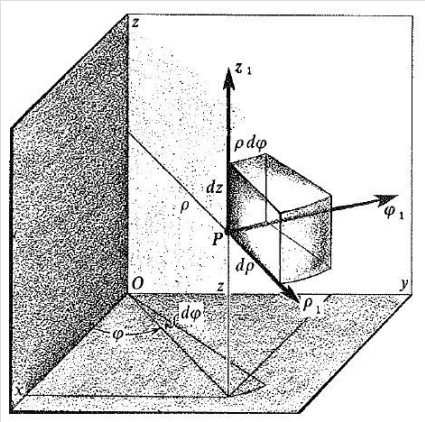
  - This divergence operation is applicable to general vector fields such as heat flux, electric current, electric field and etc.

III. Neutron Transport and Diffusion

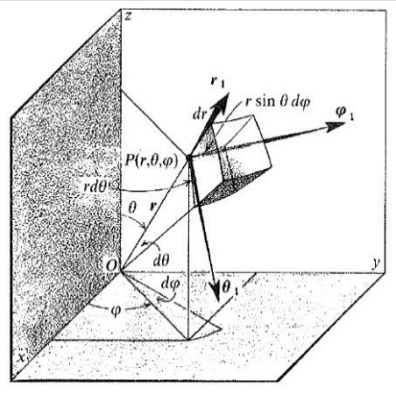
- Divergence of a Vector
  - For the other coordinate systems

$$\nabla \cdot \vec{J} = \frac{1}{r}\frac{\partial}{\partial r}(r\,J_r) + \frac{1}{r}\frac{\partial J_\varphi}{\partial \varphi} + \frac{\partial J_z}{\partial z}\,, \quad \textit{cylindrical coordinate}$$

$$\nabla \cdot \vec{J} = \frac{1}{r^2 \sin(\theta)}\left[\frac{\partial}{\partial r}(r^2 \sin(\theta)\,J_r) + \frac{\partial}{\partial \theta}(r \sin(\theta)\,J_\theta) + \frac{\partial}{\partial \varphi}(r\,J_\varphi)\right], \quad \textit{spherical}$$



Cylindrical Coordinate



Spherical Coordinate

III. Neutron Transport and Diffusion

- Divergence of Gradient of Scalar Field
  - Neutron current, J, is calculated by the Gradient operation on Neutron Flux, ϕ

$$\vec{J} = -D\nabla\phi = \vec{i}\,D\frac{\partial\phi}{\partial x} + \vec{j}\,D\frac{\partial\phi}{\partial y} + \vec{k}\,D\frac{\partial\phi}{\partial z} = \vec{i}\,J_x + \vec{j}\,J_y + \vec{k}\,J_z$$

  - The leakage rate per unit volume can be calculated by Divergence of the Neutron Current, J

$$Leakage = \nabla\cdot\vec{J}(x,y,z) = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z}$$

  - Thus, the neutron leakage is calculated from the flux distribution

$$Leakage = \nabla\cdot\vec{J}(x,y,z) = \nabla\cdot[-D\nabla\phi(x,y,z)] = -D[\frac{\partial^2\phi}{\partial^2 x} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}]$$

III. Neutron Transport and Diffusion

- Laplacian Operator
  - Divergence of Gradient of a Scalar function results in Laplacian Operation
    - Rectangular Coordinate System

$$\nabla^2\phi(x,y,z) = \frac{\partial^2\phi(x,y,z)}{\partial^2 x} + \frac{\partial^2\phi(x,y,z)}{\partial y^2} + \frac{\partial^2\phi(x,y,z)}{\partial z^2}$$

    - Cylindrical Coordinate System

$$\nabla^2\phi(r,\varphi,z) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi(r,\varphi,z)}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2\phi(r,\varphi,z)}{\partial\varphi^2} + \frac{\partial^2\phi(r,\varphi,z)}{\partial z^2}$$

    - Spherical Coordinate System

$$\nabla^2\phi(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi}{\partial r}\right) + \frac{1}{r^2 Sin(\theta)}\frac{\partial}{\partial\theta}\left(Sin(\theta)\frac{\partial\phi}{\partial\theta}\right) + \frac{1}{r^2 Sin^2(\theta)}\frac{\partial^2\phi}{\partial\varphi^2}$$

III. Neutron Transport and Diffusion

- Neutron Balance Equation
  - If we ignore the decay of neutron($T_H = 611\ sec$), the neutron balance equation can be written as

$$\frac{\partial n}{\partial t} = Production\ Rate - Absorption\ Rate - Leakage\ Rate$$

$$= \nu \Sigma_f \phi - \Sigma_a \phi - Leakage\ Rate$$

  - The number of net outgoing neutron per unit volume per unit time is given by the following equation

$$Outgoing\ neutron\ per\ unit\ volume\ per\ unit\ time$$

$$= Leakage\ Rate = \nabla \cdot J = \nabla \cdot (-D \nabla \phi) = -D \nabla^2 \phi$$

III. Neutron Transport and Diffusion

- Neutron Balance Equation
  - The neutron balance equation

$$\frac{\partial n}{\partial t} = \frac{1}{v}\frac{\partial \phi}{\partial t} = \nu \Sigma_f \phi - \Sigma_a \phi + D \nabla^2 \phi$$

  - If the neutron production and consumption is equal, the above equation becomes

$$\nabla^2 \phi + \frac{\nu \Sigma_f - \Sigma_a}{D} \phi = 0$$

  - The above equation applies to critical system and can be solved with appropriate boundary conditions

III. Neutron Transport and Diffusion

- Neutron Balance Equation
  - For the non-critical system, the neutron production term in the balance equation is divided by k_{eff} to make the right hand side of the equation zero

$$0 = \frac{1}{k_{eff}} \nu \Sigma_f \phi - \Sigma_a \phi + D \nabla^2 \phi$$

$$\nabla^2 \phi + \frac{[\frac{1}{k_{eff}} \nu \Sigma_f - \Sigma_a]}{D} \phi = \nabla^2 \phi + B^2 \phi = 0$$

  - The above eigenvalue problem can be solved with the appropriate boundary conditions and effective multiplication factor, k_{eff}, can be found

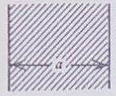
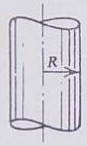
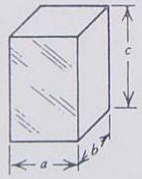
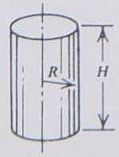
III. Neutron Transport and Diffusion

❑ Solutions of One-Group Diffusion Equation

- ➢ The flux shape for the critical, homogeneous reactors can be found by applying appropriate boundary conditions, such as $\phi_{(surface)} = 0$.

$$\nabla^2\phi(r,t) = -B_m{}^2\,\phi(r,t) \quad , \quad B.C.\ \phi(surface,t) = 0 \qquad -----(3)$$

- ➢ The flux shape for some specific reactor geometries are given below

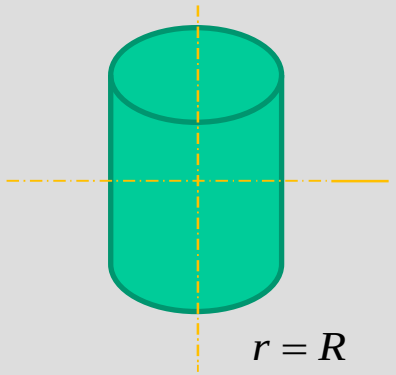
	Slab	Infinite Cylinder	Sphere	Rectangular Parallelepiped	Finite Cylinder
					
Geometric Buckling B_g^2	$\left(\frac{\pi}{a}\right)^2$	$\left(\frac{\nu_0}{R}\right)^2$	$\left(\frac{\pi}{R}\right)^2$	$\left(\frac{\pi}{a}\right)^2+\left(\frac{\pi}{b}\right)^2+\left(\frac{\pi}{c}\right)^2$	$\left(\frac{\nu_0}{R}\right)^2+\left(\frac{\pi}{H}\right)^2$
Flux profile	$\cos\frac{\pi x}{a}$	$J_0\left(\frac{\nu_0 r}{R}\right)$	$r^{-1}\sin\left(\frac{\pi r}{R}\right)$	$\cos\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{b}\right)\cos\left(\frac{\pi z}{c}\right)$	$J_0\left(\frac{\nu_0 r}{R}\right)\cos\left(\frac{\pi z}{H}\right)$

III. Neutron Transport and Diffusion

- Finite Cylindrical Reactor and Rectangular Reactor
  - Flux distribution in Finite Cylindrical Reactor

$$\phi(r,z) = A\,J_0\left(\frac{2.405\,r}{R}\right)\mathrm{Cos}\left(\frac{\pi\,z}{H}\right),$$

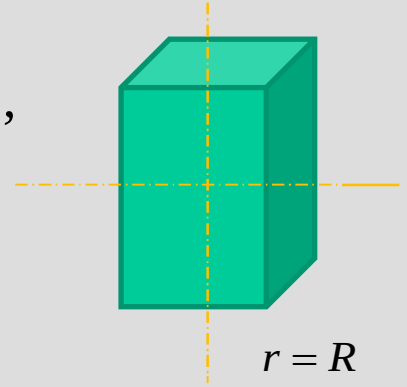
$$B^2 = \left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2$$



  - Flux distribution in Finite Rectangular Reactor

$$\phi(x,y,z) = A\,\mathrm{Cos}\left(\frac{\pi\,x}{D}\right)\mathrm{Cos}\left(\frac{\pi\,y}{W}\right)\mathrm{Cos}\left(\frac{\pi\,z}{H}\right),$$

$$B^2 = \left(\frac{\pi}{D}\right)^2 + \left(\frac{\pi}{W}\right)^2 + \left(\frac{\pi}{H}\right)^2$$

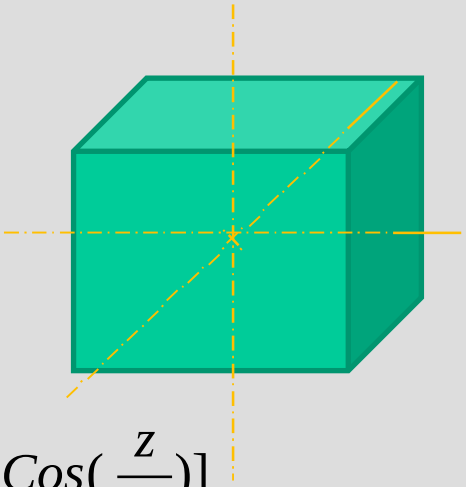


III. Neutron Transport and Diffusion

- Infinite Bare Rectangular Reactor

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}+B^2\phi=0,\quad Boundary\ Condition:\ \phi_s=0$$

  - The general solution is a multiplication of trigonometric functions



$$\phi(x)=[A\,Sin(\frac{x}{L_x})+B\,Cos(\frac{x}{L_x})]\times$$

$$[C\,Sin(\frac{y}{L_y})+D\,Cos(\frac{y}{L_y})]\times[E\,Sin(\frac{z}{L_z})+F\,Cos(\frac{z}{L_z})]$$

  - To satisfy the B.C., the sine terms are vanished and the solution becomes

$$\phi(x,y,z)=\phi_0\,Cos(\frac{x}{L_x})\,Cos(\frac{y}{L_y})\,Cos(\frac{z}{L_z}),\quad B^2=\left(\frac{\pi}{D}\right)^2+\left(\frac{\pi}{W}\right)^2+\left(\frac{\pi}{H}\right)^2$$

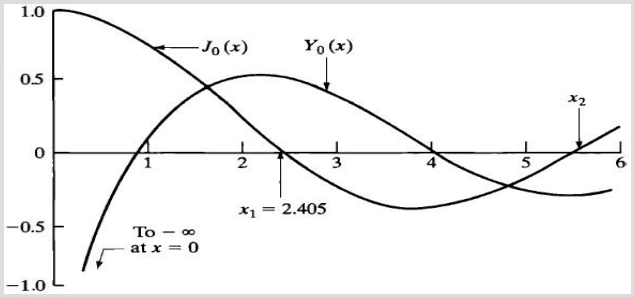
III. Neutron Transport and Diffusion

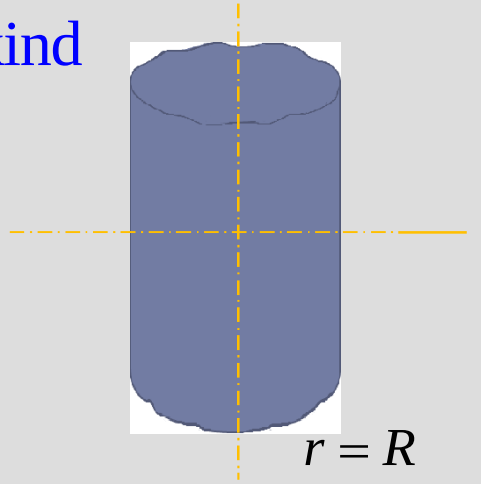
- Infinite Bare Homogeneous Cylindrical Reactor

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r} + B^2 \phi = 0, \quad \textit{Boundary Condition} : \phi(\mathrm{R}) = 0$$

  - The general solution is a combination of Bessel function of the first and second kind

$$\phi(x) = A J_0(Br) + C Y_0(Br)$$





  - To satisfy the B.C., the second term is vanished and the solution becomes

$$\phi(x) = A J_0\left(\frac{2.405\ r}{R}\right), \quad B^2 = \left(\frac{2.405}{R}\right)^2$$

III. Neutron Transport and Diffusion

- Bare Homogeneous Spherical Reactor

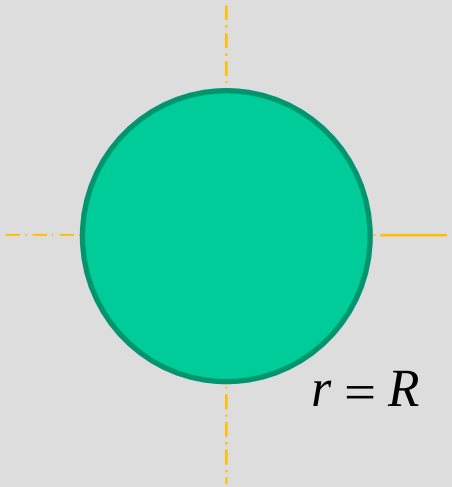
$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial \phi}{\partial r} + B^2 \phi = 0, \quad \textit{Boundary Condition} : \phi(\mathrm{R}) = 0$$

  - The solution and the buckling satisfying B.C. are

$$\phi(x) = A \frac{Sin(\frac{\pi r}{R})}{r}, \quad B^2 = \left(\frac{\pi}{R}\right)^2$$

$$P = E_R \Sigma_f \int_0^R 4\pi r^2 \phi(r)\, dr = 4\pi E_R \Sigma_f A R^2$$

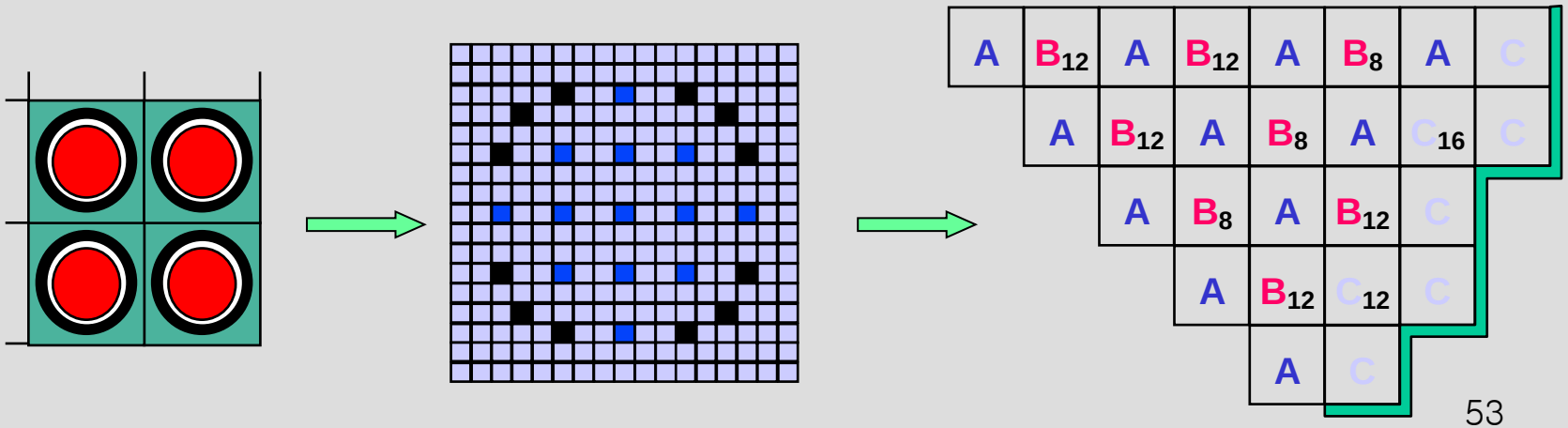
$$\phi(x) = \frac{P}{4\pi E_R \Sigma_f R^2} \frac{Sin(\frac{\pi r}{R})}{r}$$



III. Neutron Transport and Diffusion

❑ Calculation Methods

- ➢ The reactor core is composed of various materials with complex geometries and the reaction cross sections are strongly dependent on the neutron energy
- ➢ We can not solve the transport equation directly in a single step for the space and energy variable
- ➢ Space and energy condensation procedures are used while preserving the reaction rates



IV. Time Dependent Reactor

- Neutron Flux Behavior in Non-critical Reactor
  - For constant power operation, the balance between the production and consumption of neutron must be maintained
  - The reactor will become supercritical or subcritical if its properties are changed in such a way that its multiplication factor deviate from unity
  - The multiplication factor change can be result from
    - ✓ Control rod movement
    - ✓ Fuel burn up
    - ✓ Temperature changes of various reactor materials
    - ✓ Isotope production
    - ✓ Accidents
  - The time behavior of neutron density in the reactor is the necessary element to control the reactor

IV. Time Dependent Reactor

❑ Effect of the Delayed Neutron

- Most neutrons are emitted instantaneously, 10^{-17} sec after the fission reaction
- Around 0.7% of neutrons are emitted by the decay of some unstable fission product nuclides which have half-life of 0.2 ~ 55 sec, increasing life-time
- Without these delay neutrons, reactor control is impossible due to the extremely short life-time and thus rapid change in neutron population

$$l \approx 10^{-4}\,\text{sec} \Rightarrow \langle l \rangle = (1-\beta)\, l + \sum_{i=1}^{6} \beta_i \left(\frac{1}{\lambda_i} + l \right) \approx 0.1\,\text{sec}$$

- By maintaining the "prompt neutron only multiplication factor" always less then unity, we can avoid the rapid increase of neutron population
- Under the above condition, the total multiplication factor can be larger than unity by addition of delayed neutrons, and reactor control become possible

IV. Time Dependent Reactor

❑ Delayed Neutron Group

Delayed Neutron Precursor	Group	Half -life	Fraction
Br-87	Group 1	54.5 sec	0.000215
I-137 Br-88	Group 2	24.4 sec 16.3 sec	0.001424
I-138 Br-89 Rb-93, 94	Group 3	6.3 sec 4.4 sec 6 sec	0.001274
I-139 Br-90,92 Kr-93	Group 4	2.0 sec 1.6 sec 1.5 sec	0.002568
I-140	Group 5	0.5 sec	0.000748
Br, Rb, As	Group 6	0.2 sec	0.000273

IV. Time Dependent Reactor

❑ Point Kinetic Equation and Inhour Equation

- ➢ As a good approximation, we assume that the flux and the precursor distributions have the same shape but their amplitudes change
- ➢ Neutron population can be described by the Point Kinetic Equation

$$\frac{dn(t)}{dt} = \frac{(\rho(t)-\beta)}{l} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t), \qquad \rho(t) = \frac{k_{eff}-1}{k_{eff}}$$

$$\frac{dC_i(t)}{dt} = -\lambda_i C_i(t) + \frac{\beta_i}{l} n(t), \quad i = 1,2...6 \qquad \text{----------(14)}$$

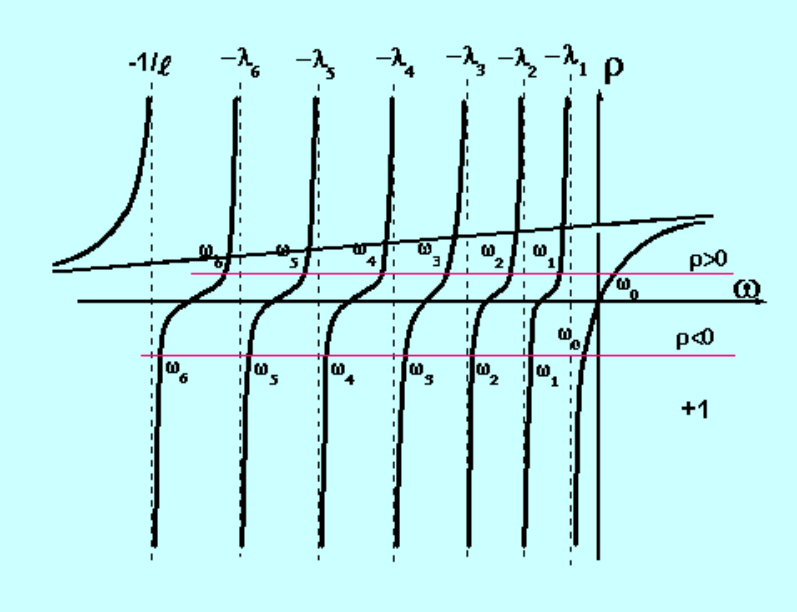
$$n(t) = \sum_{j=1}^{7} A_j \, Exp(\omega_j \, t), \qquad A_j = \frac{n_0 \, \rho_0}{\omega_j \left[\, l + \sum_{i=1}^{6} \frac{\beta_i \, \lambda_i}{(\omega_j + \lambda_i)^2} \, \right]}$$

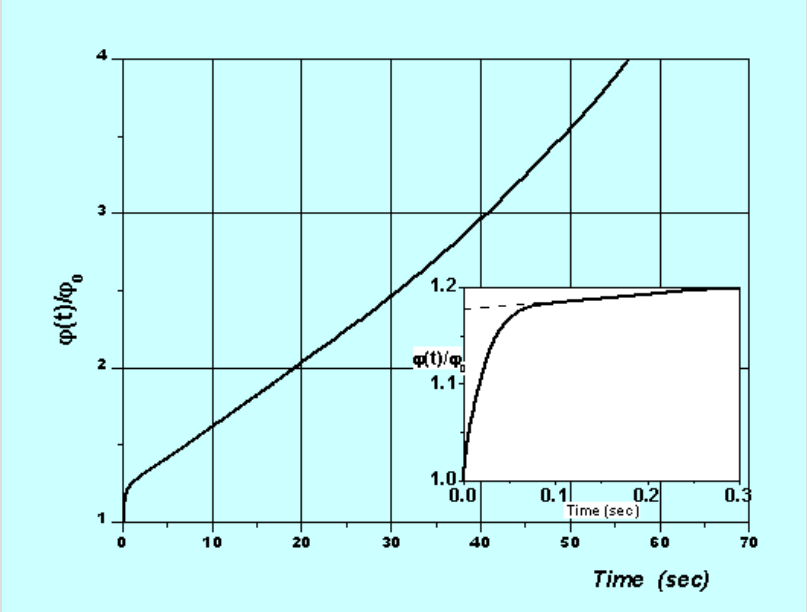
$$\rho = l\,\omega + \sum_{i=1}^{6} \frac{\beta_i \, \omega}{\omega + \lambda_i} \qquad \text{----------(15)}$$

IV. Time Dependent Reactor

❑ Point Kinetic Equation

- ➢ If we insert a step reactivity, ρ_0 , to a source free equilibrium reactor, the ω_i s , given by equation (15), can be obtained by graphically as shown below
- ➢ Neutron population change for the step reactivity insertion exhibits the prompt jump at the instant of reactivity insertion





IV. Time Dependent Reactor

❑ Application of Inverse Point Kinetic Equation

- ➢ While equation (15) shows the neutron flux behavior for the given reactivity insertion, we can calculate reactivity inversely by monitoring the neutron flux behavior
- ➢ Reactivity computer is used for reactor physics tests, which validate the physics design
  - ✓ Control Rod Worth Measurement
  - ✓ Temperature Reactivity Coefficient Measurement
  - ✓ Critical Boron Concentration Measurement

$$\rho(t) = \beta + \frac{l}{n(t)}\left(\frac{dn(t)}{dt} - \sum_{i=1}^{6}\lambda_i C_i(t)\right) \qquad --------(16)$$

$$C_i(t) = \frac{\beta_i}{l}\int_{-\infty}^{t} e^{-\lambda i\,(t-t')}\, n(t')\, dt'$$

IV. Time Dependent Reactor

- Reactivity feedback
  - The Multiplication Factor depends on the fuel temperature, moderator density and temperature, neutron absorbing nuclide density and so on
  - Because the flux behavior directly affect the above parameters, the neutron flux depend on itself as well as the other external perturbations
  - Reactor with negative reactivity coefficient has inherent stability whereas positive coefficient result in instability
  - Fuel temperature coefficient is always negative
  - Moderator temperature coefficient increases with the soluble poison density

$$Fuel\ Temperature\ Coefficient = \frac{\partial \rho}{\partial T_F}$$

$$Moderator\ Temperature\ Coefficient = \frac{\partial \rho}{\partial T_M}, \quad Void\ Coefficient = \frac{\partial \rho}{\partial V_f}$$

V. Reactor safety

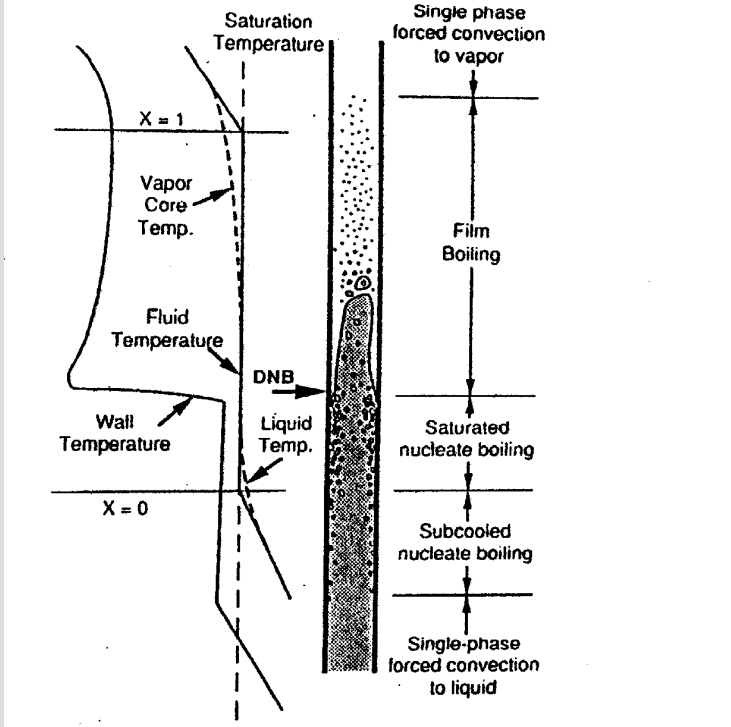
❑ Protection of Fuel Element Integrity

- ➢ Nuclear fission reaction occurs in the fuel elements, producing heat, neutrons and the fission product nuclides
- ➢ The fuel assembly experiences extremely harsh conditions such as high temperature, high pressure, high heat flux, strong radiation
- ➢ Highly radioactive fission product should be kept inside the fuel cladding
- ➢ During normal operation and anticipated transient, fuel failure should be prevented
- ➢ Under the postulated accident, coolable geometry must be maintained
- ➢ Fuel system damage is never so severe as to prevent control rod insertion when it is required

V. Reactor safety

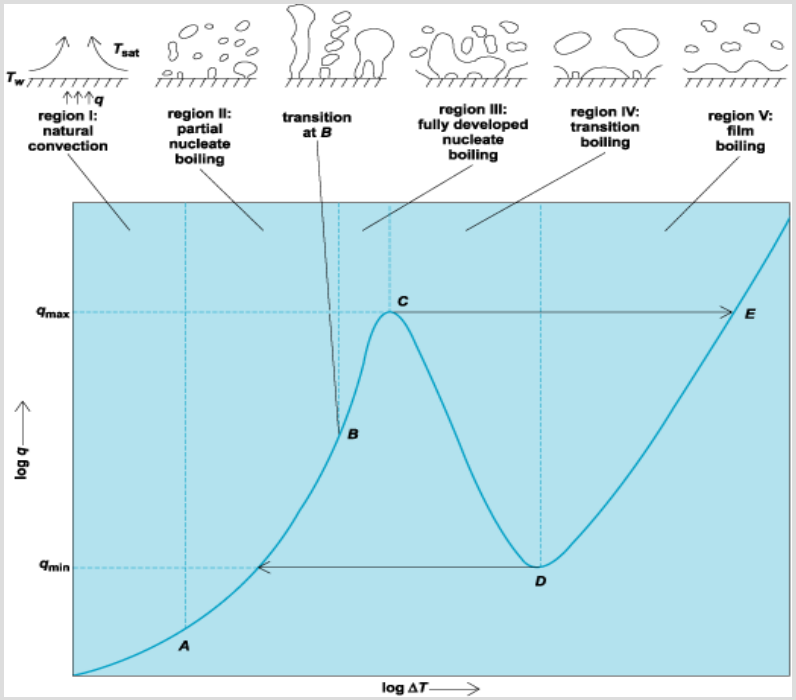
❑ Normal and Anticipated Transient Condition

- In normal operating condition of PWR, the heat produced in the fuel rods is transferred into the coolant at the nucleate boiling regime
- If the fuel rod heat flux, coolant temperature increase or coolant pressure decrease, the film boiling may occur at the fuel rod surface, which greatly reduce the heat conduction to the coolant and increase the cladding temperature



V. Reactor safety

- Normal and Anticipated Transient Condition
  - Departure from the Nucleate Boiling is prevented by reactor protection system for the anticipated transients such as loss of electric power to the reactor coolant pump



V. Reactor safety

❑ Loss of Coolant Accidents

- ➢ In the case of primary coolant system pipe rupture, the high pressure and temperature coolant is discharged from the broken loop
- ➢ Reactor protection system trip the reactor by inserting control rods into the core
- ➢ The safety injection system is ready to inject poisoned coolant but have to wait until the primary coolant system pressure drop below the injection pressure
- ➢ Due to the lack of heat removing coolant, the fuel temperature increase and integrity of the fuel cladding is challenged
- ➢ To prevent fuel failure, the following criteria is required for the performance of emergency core cooling system
  - ✓ Peak cladding temperature should not exceed 2200 F
  - ✓ Cladding oxidation thickness should be less than 17%
  - ✓ Hydrogen generation should be less than 1% of total metal water reaction
  - ✓ Coolable geometry & long term cooling should be maintained

V. Reactor safety

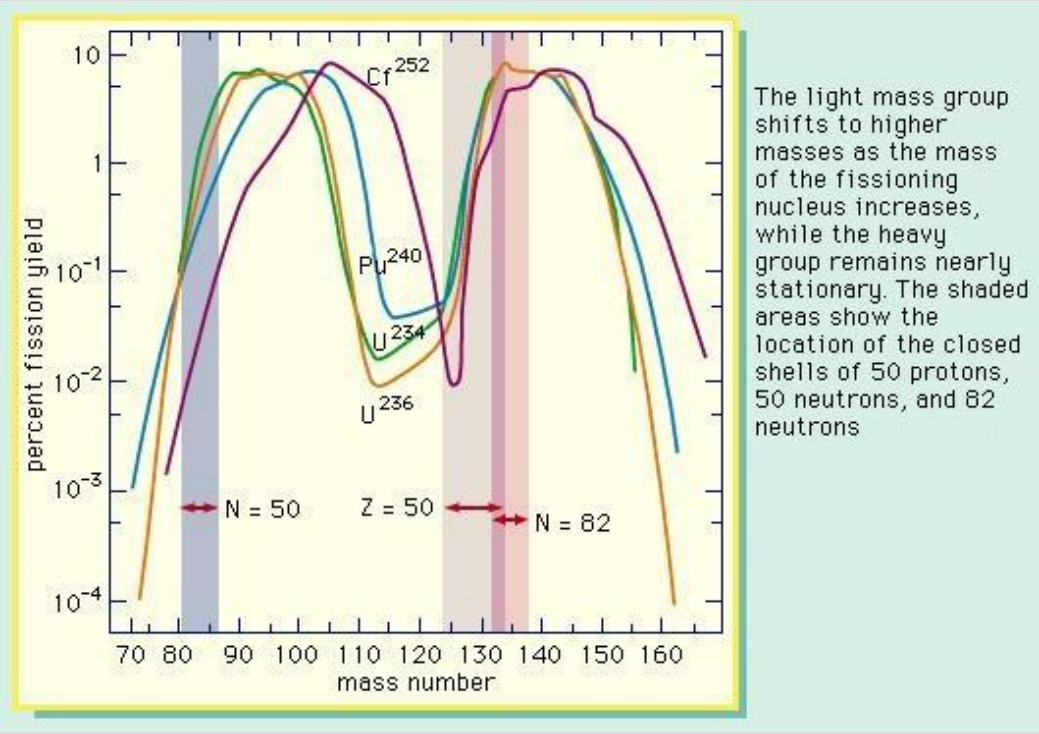
1. Production of Radioactive Nuclides

- While the nuclear fission reactions release energy, various radioactive fission products are produced
- Transuranic elements produced in the nuclear fuel by absorption of neutrons have long half-lives
- In the reactor, various structural materials are activated by radiations and become radioactive. Along with the fission product leaked from the fuel element, these radioactive materials contaminate primary coolant system and elevate the radiation level of work field

V. Reactor safety

2. Fission Product

- Yield of fission product varies with different fissile elements and incident neutron energy



V. Reactor safety

3. Decay of Fission Product

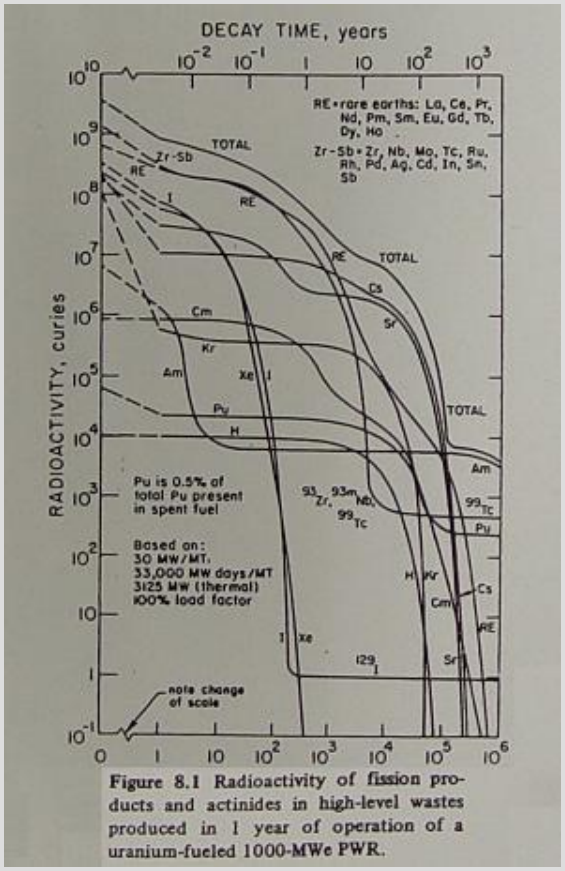


Figure 8.1 Radioactivity of fission products and actinides in high-level wastes produced in 1 year of operation of a uranium-fueled 1000-MWe PWR.

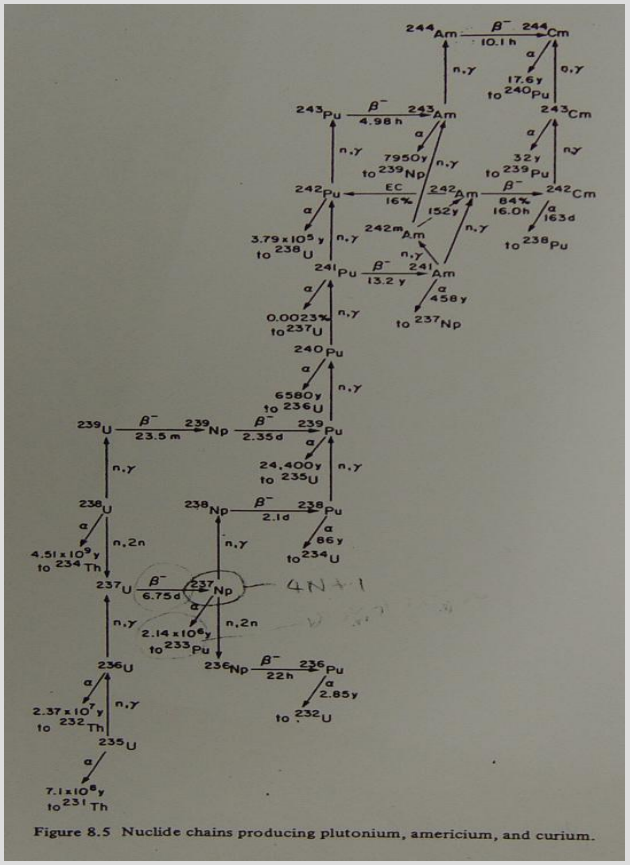


Figure 8.5 Nuclide chains producing plutonium, americium, and curium.

V. Reactor safety

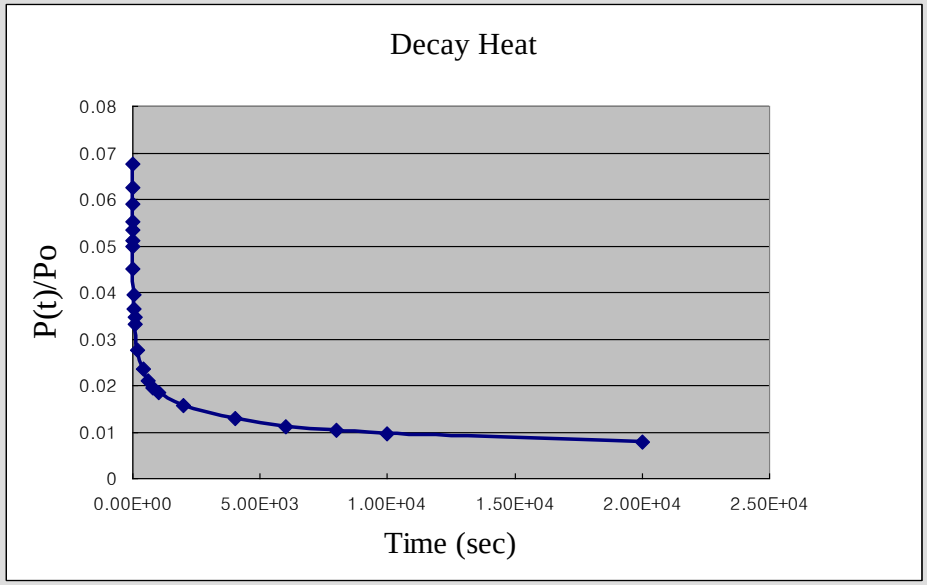
4. Multiple Barriers

- Fission products are captured in the matrix of fuel pellet which is sealed by fuel sheath
- Fission products leaked from fuel rod are mixed into the primary coolant and filtered and collected by purification system
- In case of the primary coolant system failure, radioactive materials can be contained in the reactor building

V. Reactor safety

5. Decay Heat from Fission Product

- Radioactive fission products are accumulated in the fuel rod during operation and emit radiations. These radiation energies are dissipated in the near by materials and heat up the fuel rods
- Cooling of decay heat is necessary for a prolonged period of time after shutdown



Always we keep watching
our Atomic Power



Thank You

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